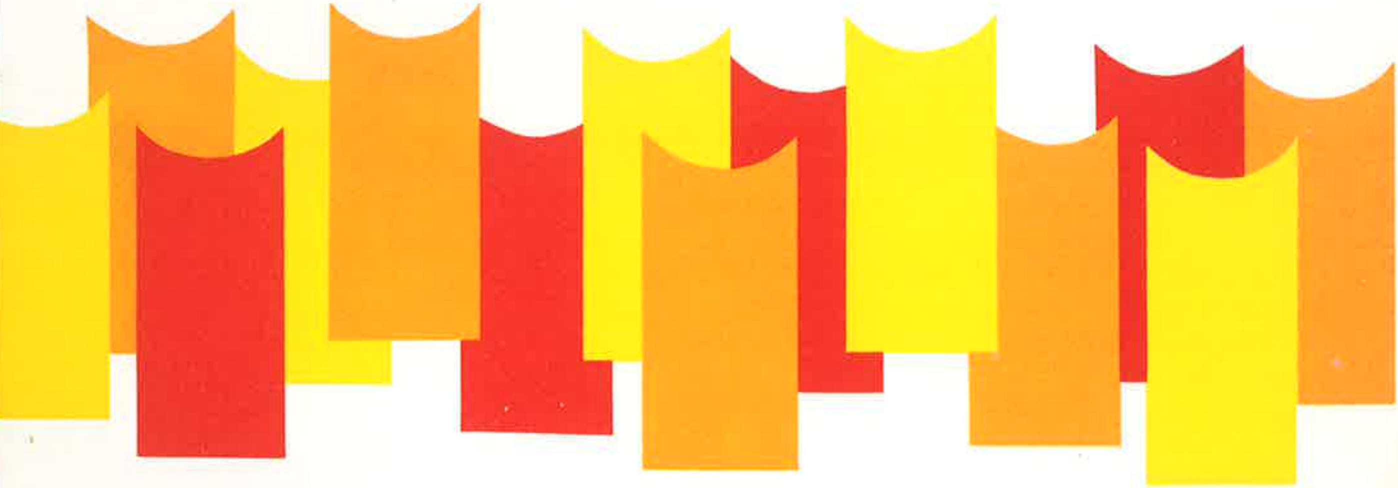
WATER & SOIL

MISCELLANEOUS PUBLICATION

No. 26

Handbook on Mixing in Rivers



NATIONAL WATER AND SOIL
CONSERVATION ORGANISATION

ISSN 0110-4705

WATER & SOIL MISCELLANEOUS PUBLICATION NO 26

HANDBOOK ON MIXING IN RIVERS

by

J. C. Rutherford
Water and Soil Division,
Ministry of Works and Development,
Hamilton, New Zealand

WELLINGTON 1981

HANDBOOK ON MIXING IN RIVERS

J. C. Rutherford
Hamilton Science Centre, Water and Soil Division,
Ministry of Works and Development, Hamilton

Water and Soil Miscellaneous Publication No. 26. 1981. 60pp.
ISSN 0110 — 4705

ABSTRACT

This handbook briefly describes the mechanisms of solute mixing in rivers and gives equations for these processes. Using these equations, with worked examples, simple techniques are given for predicting rates of mixing in rivers. The problems dealt with are those which can be solved conveniently using nomographs, programmable calculators or, at most, a small mini-computer. Use of the semi-empirical techniques described can provide a preliminary assessment of the impact of effluent on water quality.

National Library of New Zealand Cataloguing-in-Publication data
RUTHERFORD, J. C. (James Christopher), 1949-

Handbook on mixing in rivers / by J. C. Rutherford. — Wellington : Water and Soil Division Ministry of Works and Development for National Water and Soil Conservation organisation, 1981. — 1v. — (Water & soil miscellaneous publication, ISSN 0110-4705 ; no. 26)
"...briefly describes the mechanisms of solute mixing in rivers and gives equations for these processes"--Abstract.
546.225483 (628.161)
1. Water chemistry. 2. Rivers. I. Title. II. Series.

REVISED OCTOBER 1982

Published for the National Water and Soil Conservation Organisation by the Water and Soil Division, Ministry of Works and Development, P.O. Box 12-041, Wellington, New Zealand.

P. D. HASSELBERG, GOVERNMENT PRINTER, WELLINGTON, NEW ZEALAND—1983

CONTENTS

FIGURES

TABLES

LIST OF SYMBOLS

a	concentration ratio C_p/C
A	cross-sectional area
A_z, A_y	major and minor axes of concentration contours, see Figures 5.1, 5.2
b	channel width
C	concentration
$\bar{C}$	fully mixed concentration
C^*	non-dimensional concentration $C/\bar{C}$
C_p	peak concentration
C_m	minimum detectable concentration
D	molecular diffusion coefficient
D_x, D_y, D_z	dispersion coefficient in x, y and z directions
D_o, D_s	dispersion coefficient in unstratified and stratified flow
d	channel mean depth
E_x, E_y, E_z	turbulent diffusion coefficient in x, y and z directions
g	acceleration of gravity
K	von Kármán's constant ($\cong 0.4$)
k	non-dimensional length of advective zone $= LRu^*/b^2U$
L	length of advective zone
q	mass inflow rate
R	hydraulic radius
R_x	flux rate
Ri	Richardson number
S	channel slope
t	time
t_0, t_1, t_2	times when certain events occur at sites x_0, x_1, x_2
u_x, u_y, u_z	velocity in x, y and z directions
U	cross section average velocity in x direction
u^*	shear velocity $= \sqrt{gRS}$
W	total mass input
x, y, z	distance in longitudinal, vertical and tranverse directions
x^*, y^*, z^*	non-dimensional distances
x_m	distance to attain complete vertical or transverse mixing
x_0, x_1, x_2, y_0, z_0	location of sites
$x_p{}^*$	location of peak concentration
x_s, y_s	maximum length and width in which concentration exceeds a specified level
ρ	density

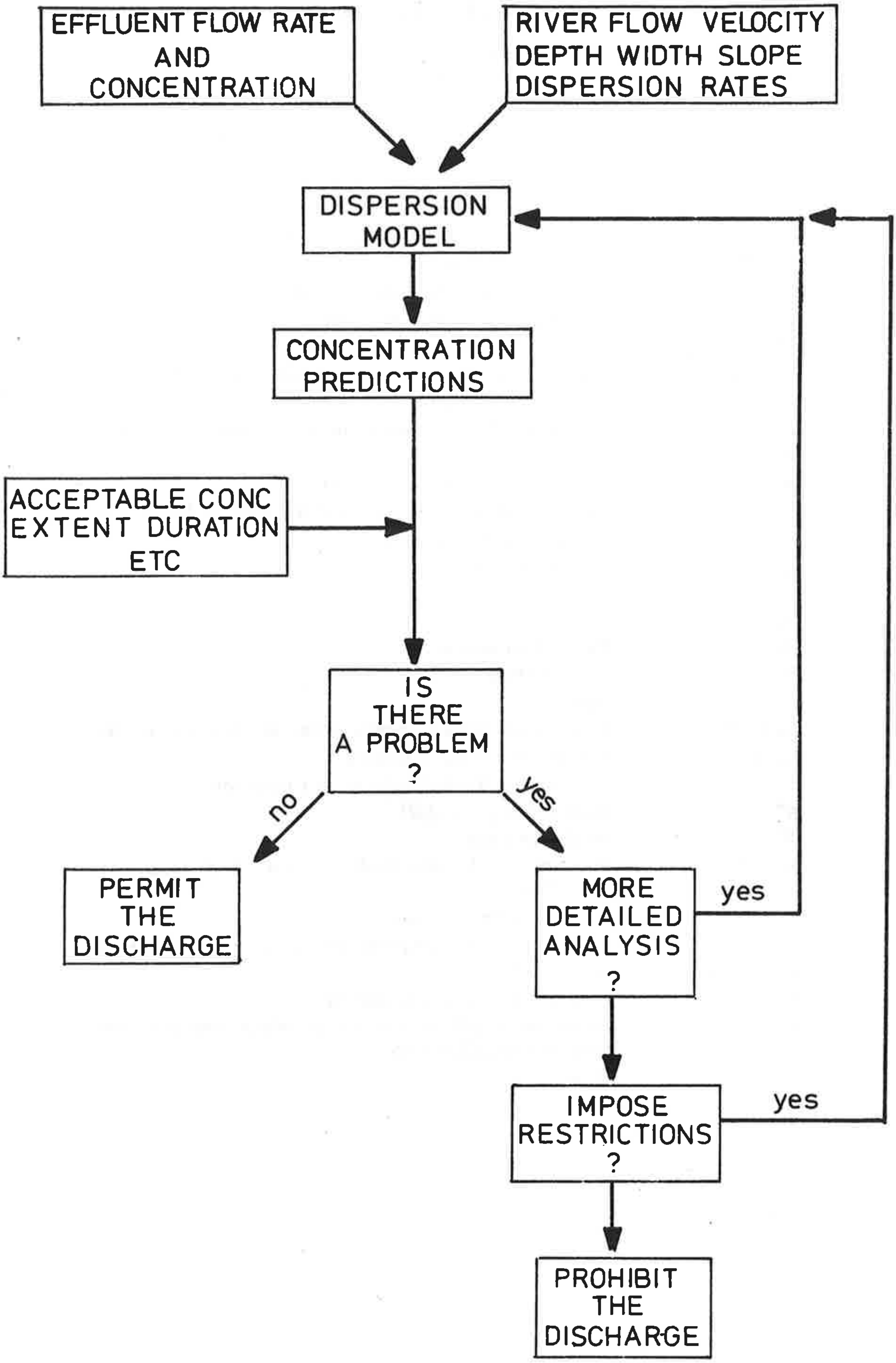


Figure 1.1 The steps and information required to assess the impact of effluents on water quality

1.0 INTRODUCTION

1.1 Scope of this handbook

One of the first steps when assessing the potential impact of an effluent on river water quality is to estimate resulting concentrations of potentially troublesome constituents. This requires knowledge of the velocities and rates of mixing in the receiving waterway. The mechanics of mixing in rivers are complex and have so far defied a complete mathematical description. There are, however, a number of semi-empirical techniques which can be used to analyse particular problems. It is the intention of this handbook to summarise simple techniques for predicting rates of mixing in rivers and to facilitate preliminary estimates of the impact of effluents on water quality.

Preliminary estimates may be sufficient to indicate whether or not an effluent will have an adverse effect on water quality, or they may indicate that further investigation, either experimental or theoretical, is justified. Figure 1.1 summarises the basic problem and the type of information required.

This handbook deals only with problems that can be solved conveniently using nomographs, programmable calculators or at most a small mini-computer. Large numerical models are not described because it is felt that these should not be employed in making a preliminary estimate of the impact of an effluent on water quality.

1.2 Mechanisms causing mixing in rivers

When material (hereinafter referred to as tracer for convenience) is discharged into a river two things happen to it. Firstly, it is carried away from the outfall by the current, a process which is termed advection; and secondly, it spreads out, a process which is termed dispersion.

In stagnant water and laminar flow, spreading is attributable to molecular motion and is called "molecular diffusion". The net transfer of tracer from a region of high concentration to a region of lower concentration proceeds at a rate proportional to the concentration gradient between the two regions. This is "Fick's Law" which in one dimension can be written mathematically

$$R_x = -D\frac{dC}{dx}$$

where R_x = transfer rate per unit area in the x direction, C = concentration, dC/dx = gradient in the x direction, and D = molecular diffusion coefficient, a constant.

In turbulent and non-uniform flow spreading proceeds at a much higher rate than in laminar flow. The reason for this is that velocity gradients act to increase concentration gradients and hence allow molecular diffusion to occur more rapidly. This is illustrated in Fig. 1.2. Such spreading is termed "dispersion" to distinguish it from "molecular diffusion". Strictly dispersion is still a molecular process but turbulence and velocity gradients greatly increase local concentration gradients and hence increase the rate at which tracer spreads.

In many situations the rate of dispersion can be approximated by Fick's Law. However, the value of D may be several orders of magnitude larger than for molecular diffusion and is highly variable. The variability arises partly because the size and intensity of turbulent eddies may vary considerably with position in the river channel, with changes in flow or location, and from one channel to another. For example the rate of dispersion can be expected to be smaller very close to the river bed (where velocity and intensity of turbulence may be small) than at mid depth. Also as the size of the tracer patch being investigated increases, the velocity gradients may change and larger eddies may become involved in mixing. Thus very close to an outfall the rate of dispersion can be expected to be smaller than it is further downstream.

1.3 Reducing the complexity of the problem

In the most general problem advection and dispersion will occur in each of the three co-ordinate directions, and the governing equations will be comparatively complex.

In many practical problems, however, the analysis can be simplified by neglecting terms which are small. This can be done: if any of the velocities is small, if any of the

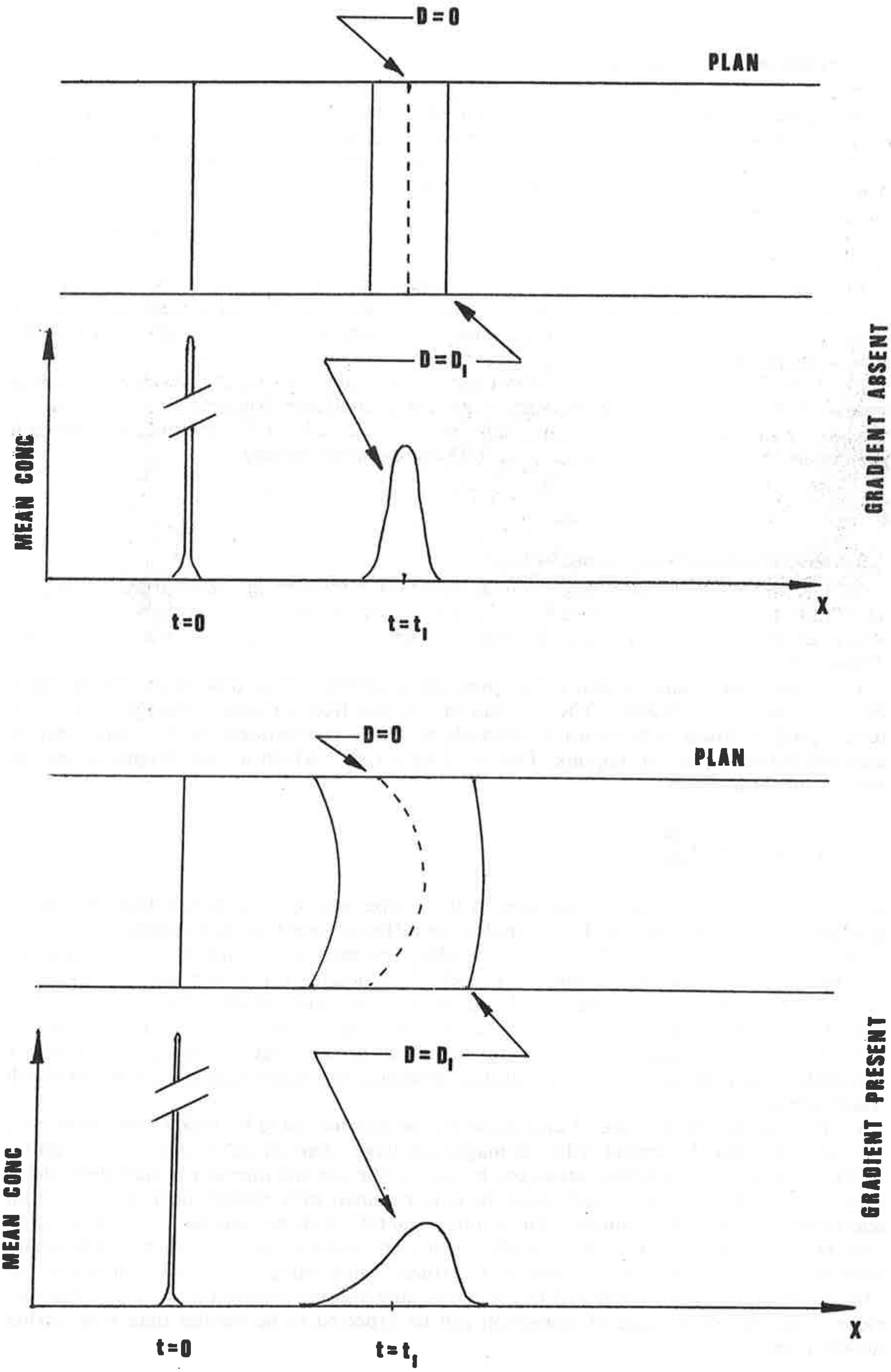


Figure 1.2 Sketch showing how a velocity gradient increases the dispersion rate.

concentration gradients is small because the tracer is far enough downstream from the outfall for the presence of channel boundaries to be felt, or if the nature of the discharge means that any concentration gradient is small.

In studying rivers we can make the following simplifications. Clearly vertical and lateral average velocities are small. Many rivers are wide but shallow, and tracer becomes well-mixed vertically before it becomes well-mixed transversely. Similarly tracer often becomes well-mixed transversely before it becomes well-mixed longitudinally. This means that vertical, transverse, and longitudinal mixing can sometimes be considered as separate one-dimensional problems. At other times longitudinal mixing can be neglected and the problem becomes two-dimensional in the vertical and transverse directions. Table 1.1 summarises the characteristics of various problems in river dispersion, and Fig. 1.3 illustrates three types of river dispersion problem.

1.4 Summary

(a) Tracer movement in a river comprises advection and dispersion.
(b) Advection is the net result of averaged velocities.
(c) Dispersion is the net result of molecular diffusion and non-uniformities in velocity.
(d) In many circumstances dispersion can be modelled approximately using Fick's Law.
(e) Although the general dispersion problem is three-dimensional, simplifications can sometimes be made to reduce the complexity of the problem.

Table 1.1 Summary of important dispersion problems in rivers and terms required to study them

Type of Source		Type of Solution		Terms required (1)		Number of dimensions
time	space (1)	(2)	(3)	advection	dispersion	
instantaneous	point source	near field	$0 < x < \frac{U}{2} d^2/D_y$	x (4)	x, y, z	3
		mid field	$\frac{U}{2} d^2/D_y < x < \frac{U}{2} b^2/D_z$	x	x, z	2
		far field	$\frac{U}{2} b^2/D_z < x < \infty$	x	x	1
steady	point source	near field	$0 < x < \frac{U}{2} d^2/D_y$	x (4)	y, z	3 (6)
		mid field	$\frac{U}{2} d^2/D_y < x < \frac{U}{2} b^2/D_z$	x	z	2 (6)
		far field	$\frac{U}{2} b^2/D_z < x < \infty$	- (5)	- (5)	0 (5)
instantaneous	z-line source	near field	$0 < x < \frac{U}{2} d^2/D_y$	x	x, y	2
		far field	$\frac{U}{2} d^2/D_y < x < \infty$	x	x	1
steady	z-line source	near field	$0 < x < \frac{U}{2} d^2/D_y$	x	y	2 (6)
		far field	$\frac{U}{2} d^2/D_y < x < \infty$	- (5)	- (5)	0 (5)
instantaneous	y-line source	near field	$0 < x < \frac{U}{2} b^2/D_z$	x	x, z	2
		far field	$\frac{U}{2} b^2/D_z < x < \infty$	x	x	1
steady	y-line source	near field	$0 < x < \frac{U}{2} b^2/D_z$	x	z	2 (6)
		far field	$\frac{U}{2} b^2/D_z < x < \infty$	- (5)	- (5)	0 (5)

NOTES:

(1) co-ordinate directions are shown in Fig. 1.3

(2) near field = very close to the outfall, mid field = moderately and far field = some considerable distance away

(3) D_x, D_y, D_z are dispersion coefficients and U = mean velocity (see equations 2.12 and 3.9 in text)

(4) on a very small scale, y, z advection may be present in the prototype

(5) concentration is constant (fully mixed)

(6) the dimensionality can be reduced by one if the co-ordinate system used travels downstream at mean velocity

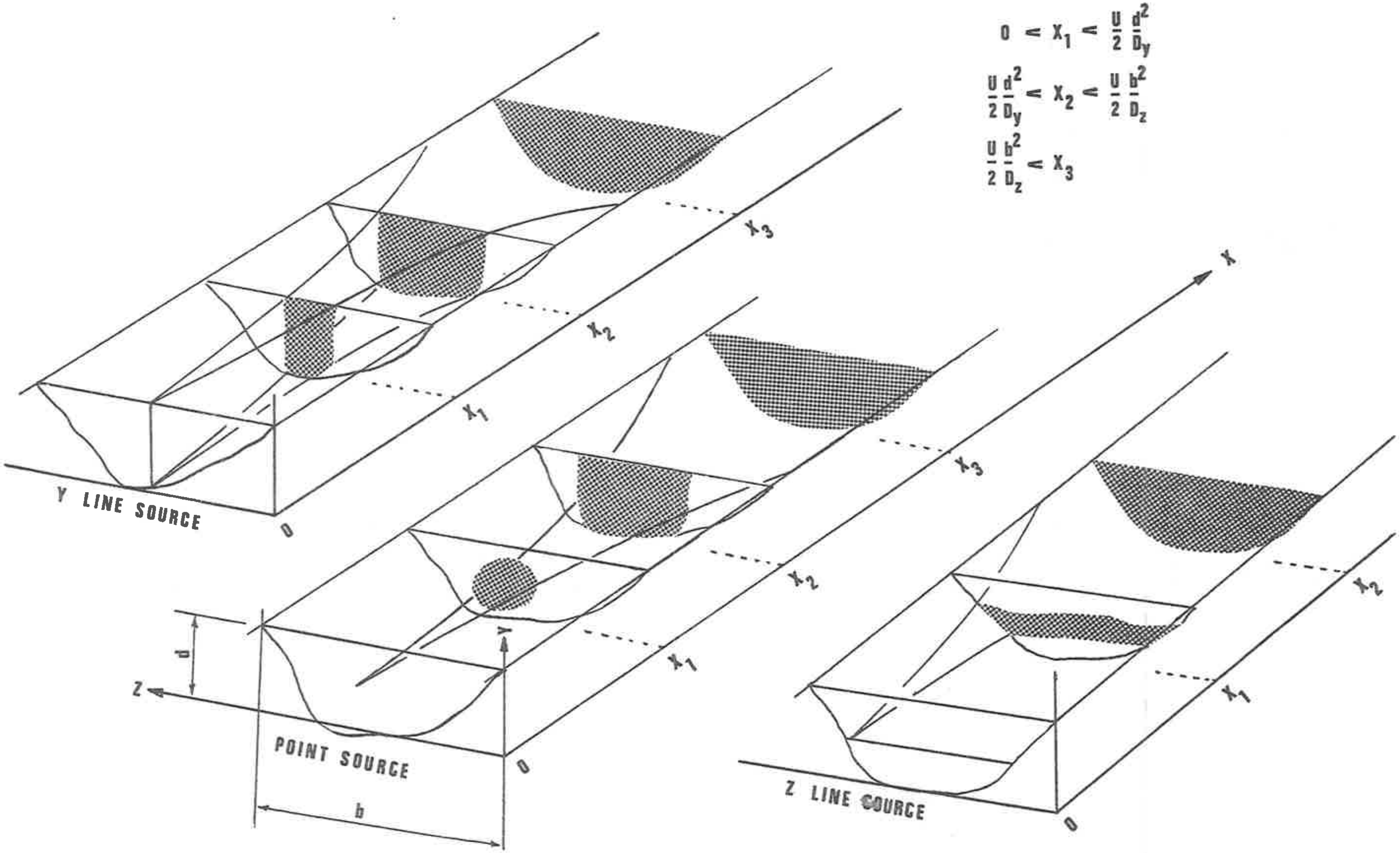


Figure 1.3 Sketch of three types of river dispersion problem.

2.0 VERTICAL MIXING

2.1 Mechanisms causing vertical mixing

In channels with no secondary circulations, the principal mechanism causing vertical mixing is turbulence generated by velocity shear. Theoretical work by Elder (1959) indicates that in such channels the dispersion coefficient varies parabolically with depth, and depends on both depth and shear velocity.

$$D_y(y) = \frac{y}{d}\left(1 - \frac{y}{d}\right) K\, d\, u^* \tag{2.1}$$

where D_y = vertical dispersion coefficient, d = depth of flow, K = von Kármán's constant (= 0.4), and u^* = shear velocity = $\sqrt{gdS}$ where S = channel slope. This form has been confirmed in laboratory and field studies. For many practical problems the depth average is used (Fischer 1973).

$$D_y = 0.067\, d\, u^* \tag{2.2}$$

Vertical secondary circulations can be expected to increase the rate of vertical mixing in natural channels. Few data are available to quantify their effect, but it appears that

$$0.067 < D_y/d\, u^* < 0.33 \tag{2.3}$$

Table 2.1 summarises some reported values of D_y.

Table 2.1 Reported vertical dispersion coefficients

Reference	Channel	D_y $\text{cm}^2.\text{s}^{-1}$	D_y/du^* —	D_y/dU —
Elder (1959)	theoretical analysis	-	$\frac{y}{d}\left(1 - \frac{y}{d}\right)K$	-
Fischer (1973)	laboratory flume	-	0.067(1)	-
Fischer (1976)	James Estuary	-	-	2.9 x 10^{-4}(2)
Fischer (1976)	Kennebec Estuary	50–650	-	-
Fischer (1976)	Mersey River	5-71(3) 500 (4)	-	-

NOTES:
(1) depth mean value
(2) augmented by wind induced surface waves
(3) measured in stratified flow
(4) estimated for non-stratified flow

2.2 Effects of density stratification

In tidal channels density stratification often occurs with saline (more dense) water underlying fresh (less dense) water. In such stratification vertical water movement is suppressed by the density gradient and the coefficient of vertical dispersion is greatly reduced. Our understanding of the processes involved is poor and it is difficult to make accurate predictions of vertical mixing rates in stratified flow. The method outlined below must, therefore, be regarded as approximate.

The "strength" of the stratification is quantified by the non-dimensional gradient Richardson number, Ri, which is the ratio

$$\frac{\text{potential energy required for mixing}}{\text{kinetic energy available to cause mixing}}$$

$$Ri = g \frac{\partial \rho}{\partial y} / \rho\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.4}$$

where g = acceleration of gravity, $\rho(y)$ = density, $u(y)$ = longitudinal velocity and y = depth.

The two gradients in equation 2.4 can be estimated satisfactorily from the slopes of straight lines fitted to density and velocity versus depth profiles measured in the field. In a tidal channel it is desirable to calculate average values of the gradients over the tidal period from, say, hourly measurements (see worked example 2.6.6).

An empirical relationship used to quantify the reduction in dispersion coefficient is

$$D_S = D_O\,(1 + a\,Ri)^b \tag{2.5}$$

where D_S and D_O = vertical dispersion coefficient in stratified and unstratified flow respectively and a and b = constants estimated variously as shown in Table 2.2.

Table 2.2 Coefficients describing the effects of stratification on the vertical dispersion coefficient (after Fischer 1976)

a	b
3.33	- 1.50
0.276	- 2.00

These models differ considerably at high Richardson numbers, (i.e. in highly stratified flow), but they indicate that vertical dispersion coefficients are reduced by a factor of 2–10 at $Ri = 1$ and 15-200 at $Ri = 10$. Clearly, therefore, the methods outlined here should only be used to make preliminary estimates of vertical mixing in stratified flow and field tests together with more detailed modelling should be undertaken to confirm findings.

2.3 Vertical mixing downstream from a steady uniform transverse line-source

The first problem considered is to predict tracer concentrations downstream from a steady uniform transverse line-source such as a perforated pipe which extends across the entire channel width. In this problem transverse concentration gradients are negligible because of the uniform line-source. Longitudinal gradients are also negligible because the source is steady. Thus the problem simplifies to become quasi one-dimensional (see Table 1.1). Since the analysis is only approximate, the velocity and dispersion coefficient are taken to be uniform over the depth as a rough approximation to turbulent flow. This simplification means that concentration estimates below outfalls on the river bed may be poorly estimated as explained below.

Figure 2.1 shows lines of equal concentration downstream from transverse line sources located at three different depths. Variables are expressed in non-dimensional form so that many combinations of parameters appear on the same graph.

$$C^* = C/\overline{C} = CUbd/q \tag{2.6}$$

$$y^* = y/d \tag{2.7}$$

$$x^* = x\,D_y/Ud^2 \tag{2.8}$$

where C^*, y^* and x^* = non-dimensional concentration, vertical displacement and downstream displacement respectively, C = concentration, $\overline{C}$ = fully mixed concentration, U = mean velocity, D_y = depth averaged vertical dispersion coefficient, d = river depth (the mean depth should be used here if the channel is irregular), b = river width and q = tracer mass inflow rate. The bed and water surface are located at $y^* = 0$ and $y^* = 1$ but the problem is symmetrical in y since the flow velocity is assumed uniform.

Clearly

$$0 < y^* < 1 \tag{2.9}$$

and

$$C^* = 1 \tag{2.10}$$

a long way below the outfall. The regions to the left of the $C^* = 0.001$ contour do not contain any tracer, while in the region to the right of the $C^* = 1.01$ and 0.99 contours the tracer is fully mixed.

Figure 2.1a may overestimate the rate of dispersion downstream from an outfall on the bed of a rough natural channel for the following reasons:

(i) the velocity very close to a boundary is small and hence concentrations will be higher locally than expected;

(ii) the value of the dispersion coefficient may be quite low very close to the boundary because the scale and intensity of turbulence are small (see equation 2.1);

(iii) irregularities in the bed, "dead zones", trap tracer and cause locally high concentrations.

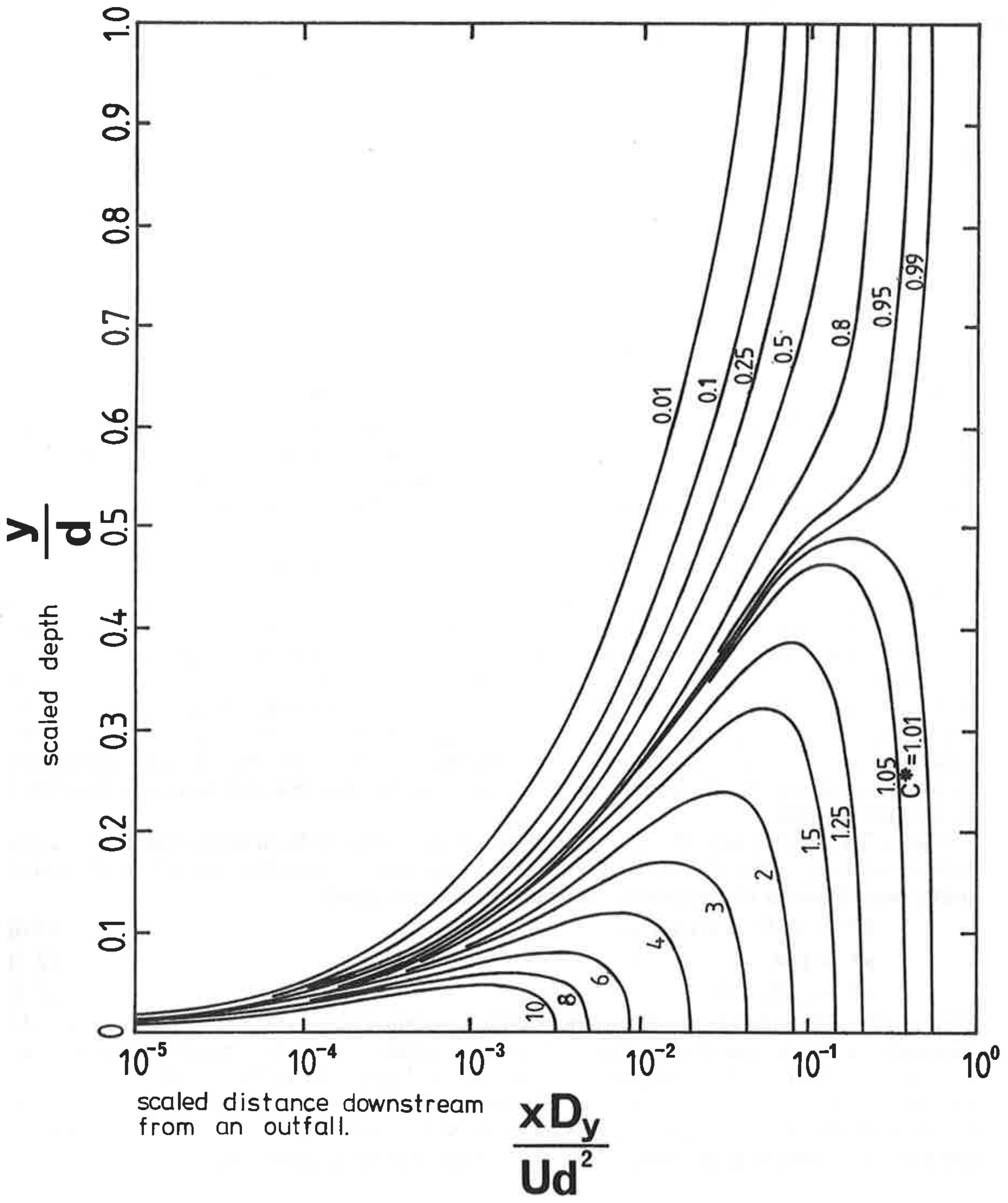


Figure 2.1a Concentration contours downstream from a steady transverse line source located on the channel bed. (See text for caveat on use of this figure; contours are of equal scaled concentration.)

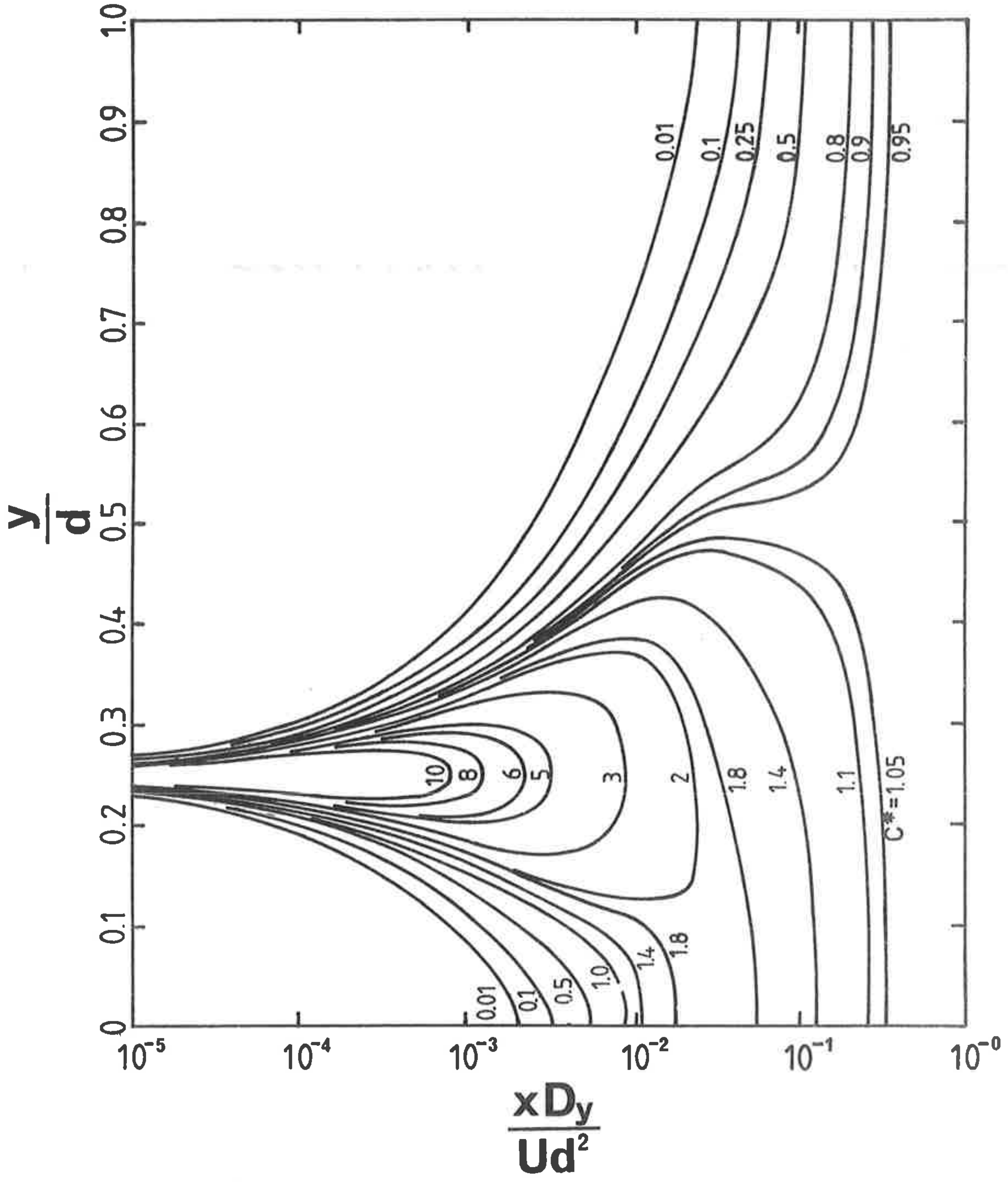


Figure 2.1b Concentration contours downstream from a steady transverse line source located at three-quarters depth.

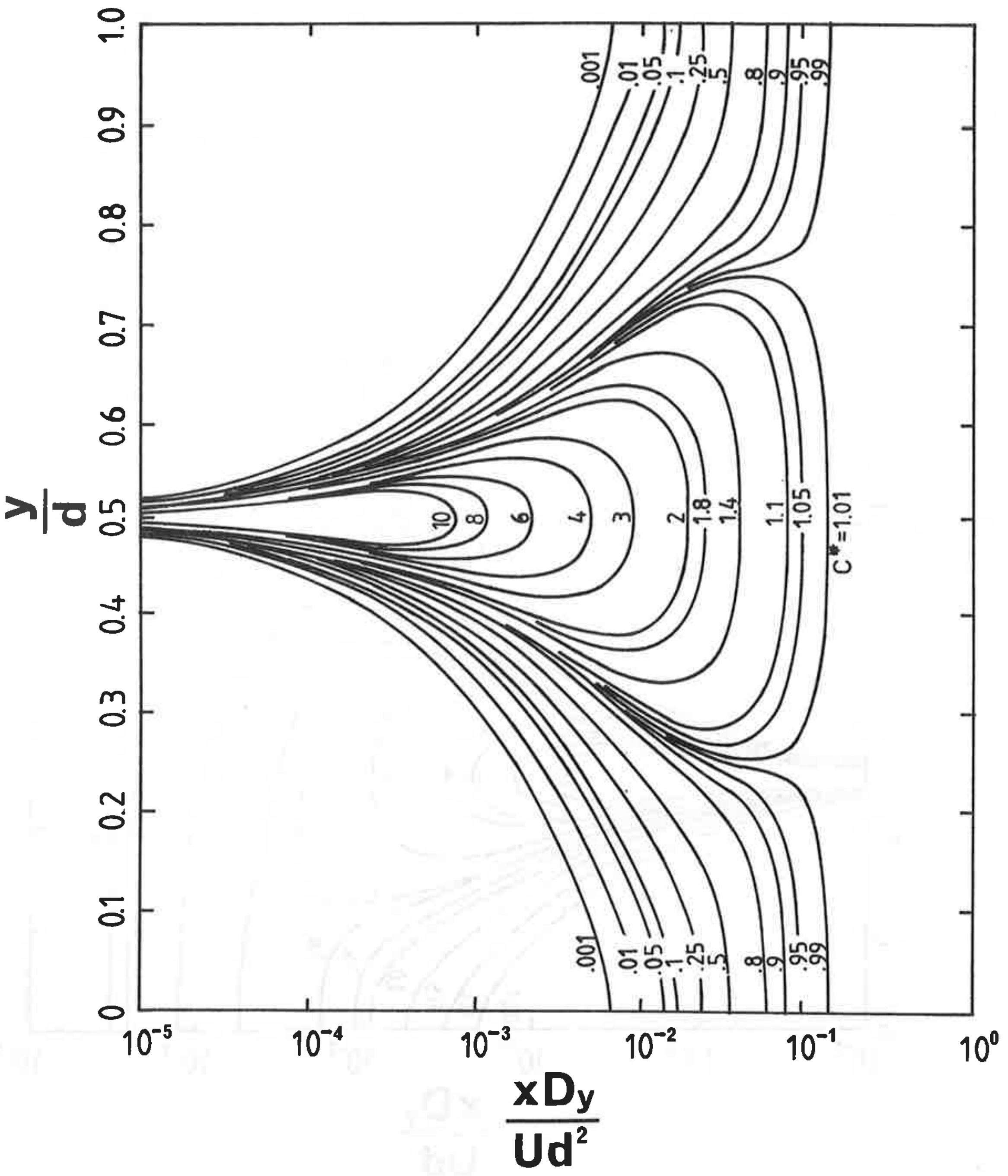


Figure 2.1c Concentration contours downstream from a steady transverse line source located at mid depth.

It is suggested that in order to make a preliminary estimate of mixing in this situation the cross-section average velocity be used for U but a very conservative estimate of dispersion coefficient be selected (say 1–10% of the depth average value) for D_y. Ideally these preliminary estimates should be checked by field tests or more sophisticated modelling (see worked example 2.6.7).

From Fig. 2.1c it can be seen that complete mixing is attained within a distance

$$x_m \cong 0.1\ Ud^2/D_y \tag{2.11}$$

downstream from an outfall located at mid-depth (Shen 1973). Vertical mixing occurs more slowly from an outfall located at the bed or on the surface because tracer cannot disperse across boundaries. Thus from Fig. 2.1a mixing is attained within a distance

$$x_m \cong 0.4\ Ud^2/D_y \tag{2.12}$$

downstream from an outfall on the bed or at the surface (Shen 1973). As noted earlier low mixing rates may be encountered near the bed and a conservative estimate of D_y should be used in equation 2.12.

Figure 2.1 also indicates the length and width of tracer plume in which concentrations exceed a specified level (see Section 2.6 for worked examples).

2.4 Vertical mixing downstream from a steady point source

A more complex problem is to predict concentrations downstream from a steady point source such as a single port outlet. Clearly both transverse and vertical dispersion are important close to the outfall although longitudinal disperson can be neglected if the source is steady (Holly 1975). Thus the problem becomes quasi two-dimensional (see Table 1.1).

Three-dimensional cigar shaped surfaces of equal concentration are encountered below a point source. It is usually sufficient when studying vertical mixing, however, to consider contours of equal concentration on a vertical (x–y) plane which passes through the outfall. Figure 2.2 shows such contours for point sources located at three depths. The outfall is located near the middle of a channel with an aspect ratio, b/d, of 50. The ratio of transverse/vertical dispersion coefficients is taken as 3, a value commonly found in rivers (see Section 3). The non-dimensional variables defined in equations 2.6 to 2.8 are used again. Here, however, d should be taken as the depth near the outfall if the channel is irregular.

As was noted earlier, a low rate of dispersion occurs near boundaries and a conservative estimate of D_y should be used in Fig. 2.2a for an outfall on the river bed.

Clearly higher concentrations are found immediately below a point-source than below a line-source of the same total output. However, tracer becomes vertically well-mixed at much the same distance below point and line sources, and equations 2.11 and 2.12 still apply. Thereafter, transverse mixing dominates and eventually mixes tracer throughout the river channel.

If the outlet is located at either bank the theory indicates that concentrations are exactly twice those shown in Fig. 2.2 with the exception of the $C^* = 1.01$ contour which moves slightly to the right (see equations 3.8 and 3.9). This does not affect the distance within which complete vertical mixing is attained. In practice, however, the rate of dispersion near a boundary is low and a conservative estimate of D_y (say 1–10% of the average) should be used.

Figure 2.2 can be used to determine the length and width of plumes in which concentrations exceed a specified level.

2.5 Vertical mixing below a steady multi-point source

Figures 2.1 and 2.2 are valid for outfalls at a particular depth but in some cases effluent may be released over a finite depth via a diffuser. This problem can be solved with the information presented earlier. It is assumed that the outfall comprises several point sources. Concentration contours are then determined for each point source separately using Fig. 2.1 and 2.2. Finally concentrations are added to produce the concentration contours for the multi-point source (see Section 2.6 for a worked example).

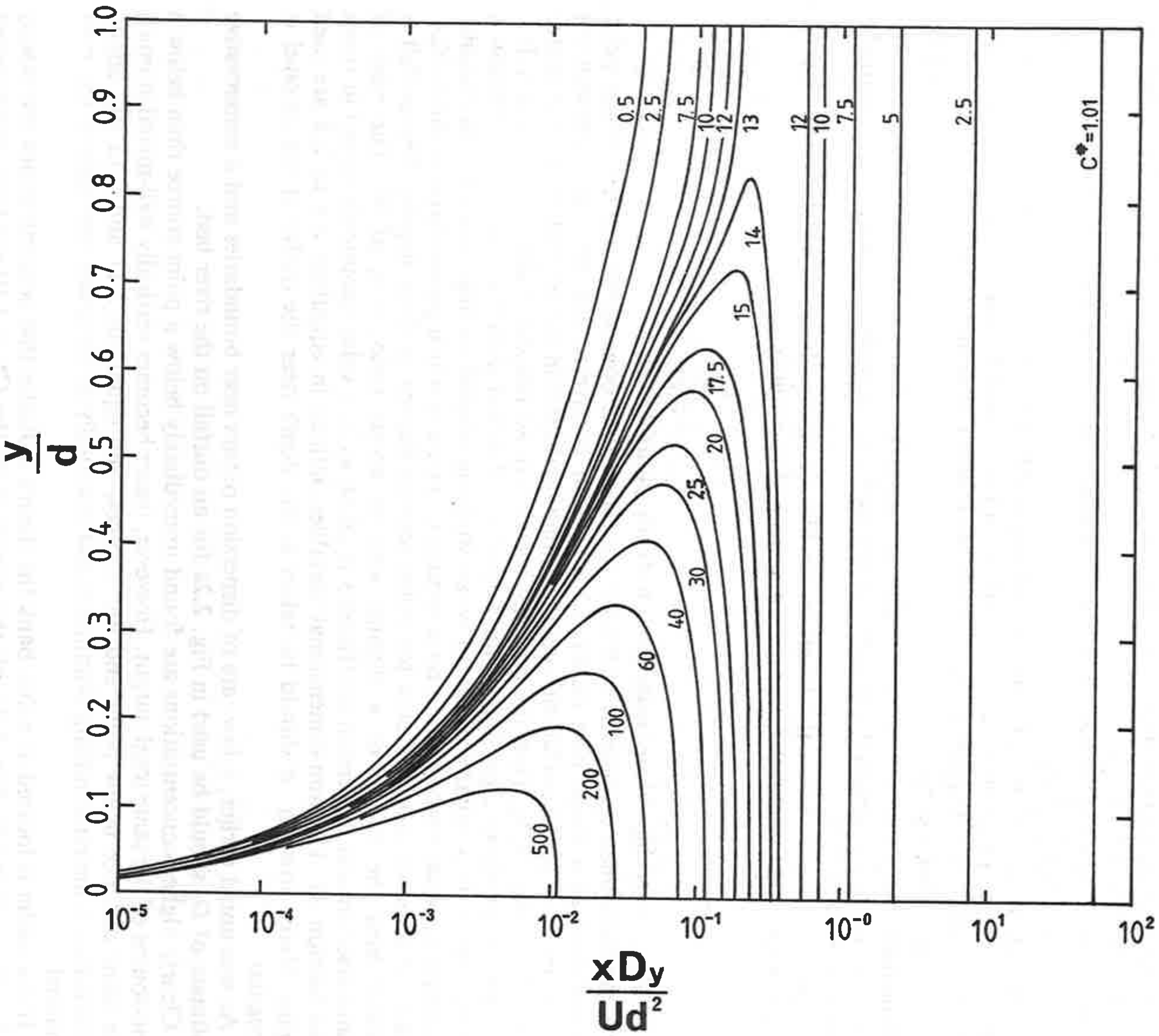


Figure 2.2a Concentration contours downstream from a steady point source located in mid channel on the bed. (See text for caveat on use of this figure.)

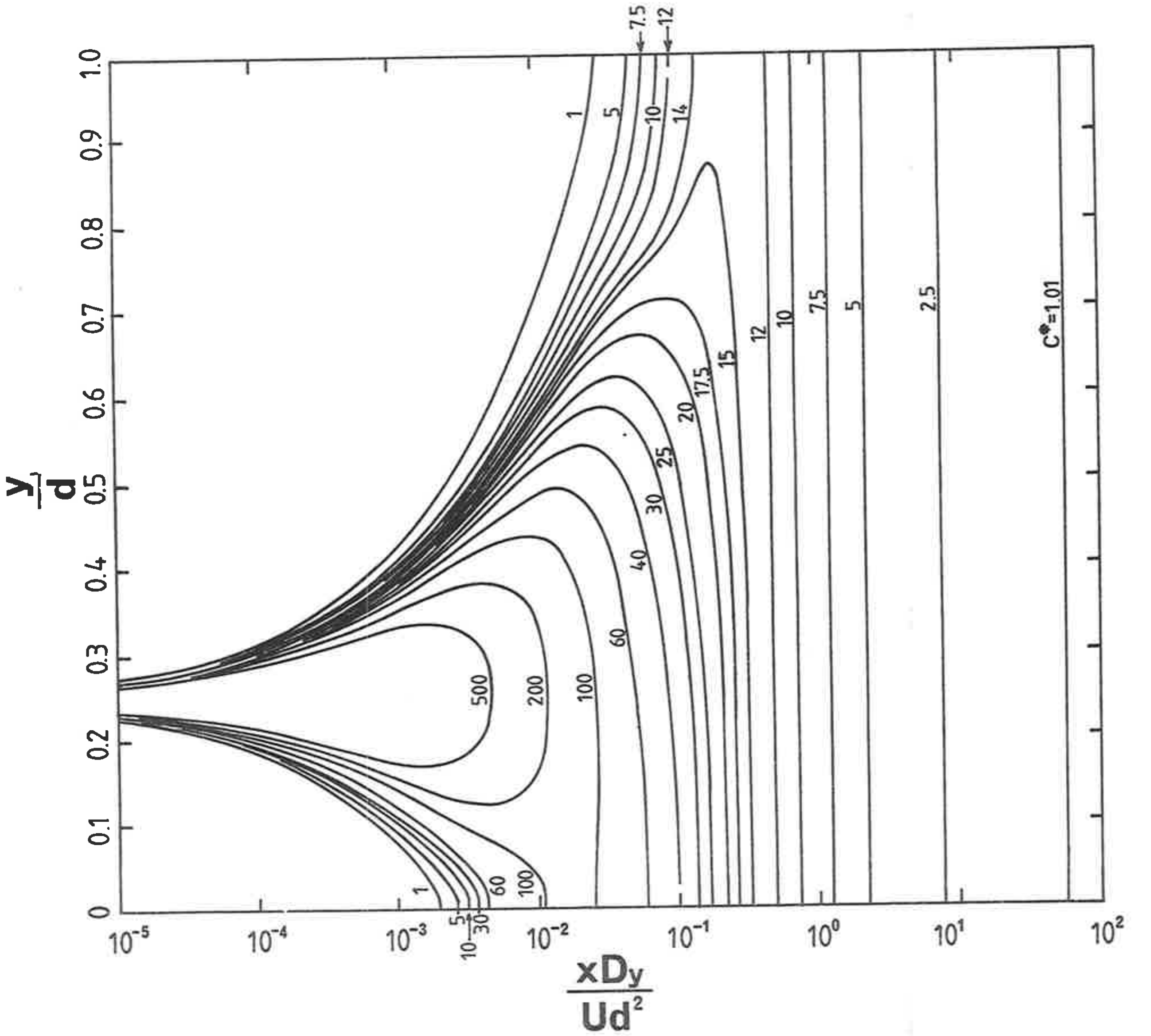


Figure 2.2b Concentration contours downstream from a steady point source located in mid channel at three-quarters depth.

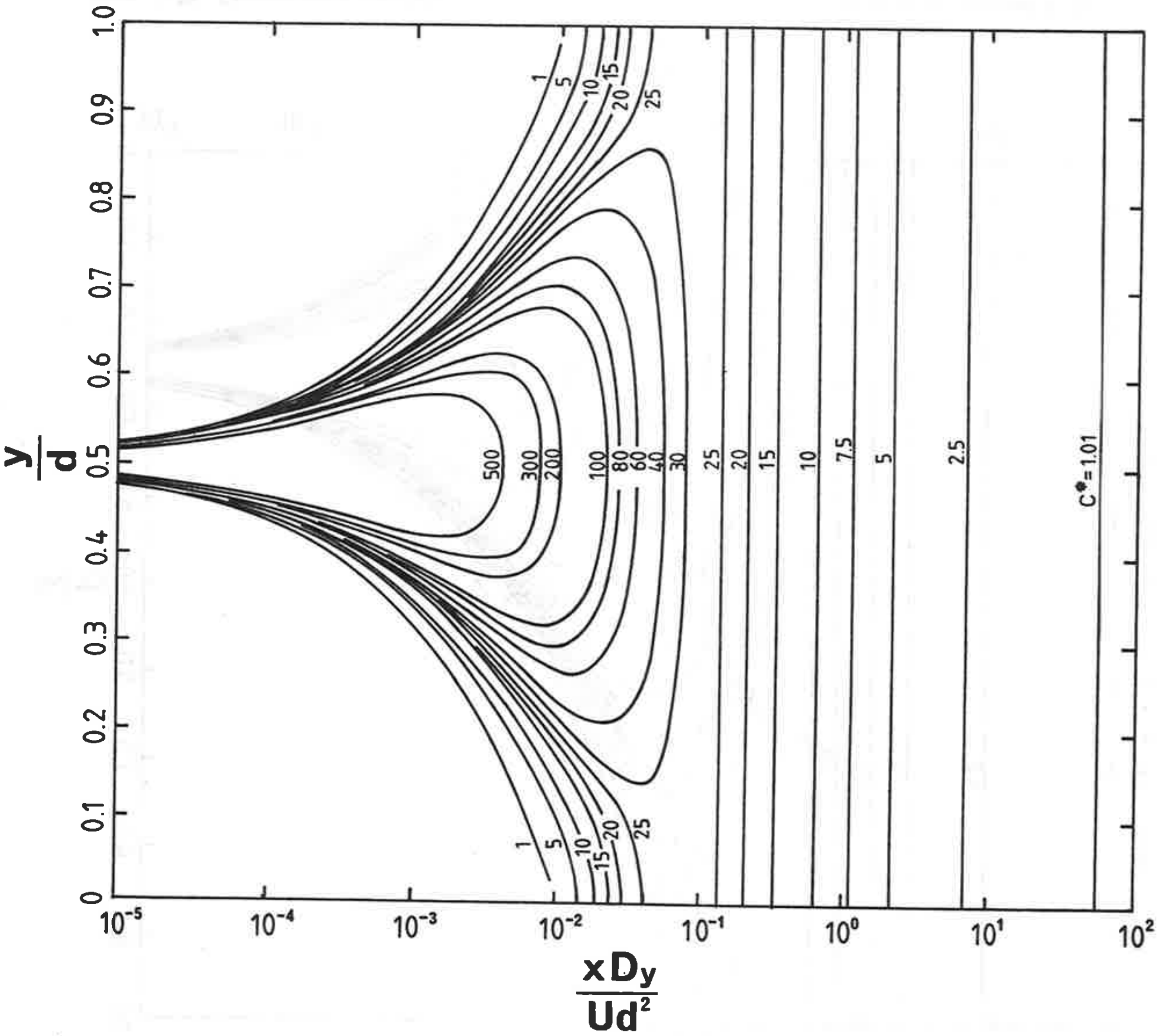


Figure 2.2c Concentration contours downstream from a steady point source located in mid channel at mid depth.

2.6 Worked examples

Assume a channel with

depth $d = 1$ m
width $b = 10$ m
slope $S = 10^{-4}$
velocity $U = 1$ m.s^{-1}

Then shear velocity $u^* = (9.81 \times 1 \times 10^{-4})^{1/2} = 0.0313$ m.s^{-1}.

Example 2.6.1 Select values of vertical dispersion coefficient assuming the channel is

(a) man-made, smooth and uniform
(b) natural but fairly uniform
(c) irregular

From equation 2.3 (a) $D_y = 0.067du^* = 20$ cm^2.s^{-1}
(b) $D_y = 0.15du^* = 50$ cm^2.s^{-1}
(c) $Dy = 0.33du^* = 100$ cm^2.s^{-1}

Example 2.6.2 Taking $Dy = 20$ cm^2.s^{-1}, determine the distance downstream from the outfall required for complete mixing for

(i) a point source, and
(ii) a transverse line source

located at (a) the surface
(b) mid-depth

(i) *Point source* (a) From equation 2.12 $x_m = 0.4Ud^2/D_y = 200$ m
(b) From equation 2.11 $x_m = 0.1 \quad Ud^2/D_y = 50$
(ii) *Line source* The results from (i) apply.

Example 2.6.3 For an outfall at the surface with mass flow 20 g.s^{-1} determine

(i) total length, x_s, and
(ii) maximum spread, y_s,

of the plume in which concentrations exceed 10 g.m^{-3}.

Consider two cases:
(a) a transverse line source, and
(b) a point source

Take $D_y = 20$ cm^2.s^{-1}.

Fully mixed concentration $\bar{C} = q/Udb = 20/1x1x10 = 2$ g.m^{-3}.
Then from equation 2.6, $C^* = 5$.

(a) *Line source*

(i) From Fig. 2.1a, the $C^* = 5$ contour extends a maximum distance of $x_s D_y/Ud^2 = 0.014$ downstream. Thus $x_s = 6.9$ m.

(ii) From Fig. 2.1a the maximum spread of the $C^* = 5$ contour is $y_s/d = 0.10$. Thus $y_s = 0.1$ m.

(b) *Point source*

From Fig. 2.2a (i) $x_s D_y/Ud^2 = 2.13 \quad x_s = 1100$ m
(ii) $y_s/d = 1 \quad y_s = 1$ m

Example 2.6.4 Repeat example 2.6.3 for an outfall at mid-depth.

(a) *Line source*
From Fig. 2.1c (i) $x_s D_y/Ud^2 = 0.003 \quad x_s = 1.5$ m
(ii) $y_s/d = 0.1 \quad y_s = 0.1$ m

(b) *Point source*
From Fig. 2.2c (i) $x_s D_y/Ud^2 = 2.13 \quad x_s = 1100$ m
(ii) $y_s/d = 1.0 \quad y_s = 1$ m

Example 2.6.5 For a discharge of mass flow 20 g.s^{-1} from
(i) a line source at mid-depth, and
(ii) a plate diffuser between 0.4 and 0.6 m depth stretching across the entire channel width,

compare the length x_S and spread y_S of plumes in which concentrations exceed
(a) 4 g.m^{-3}
(b) 8 g.m^{-3}

Take D_y = 20 cm^2.s^{-1}.

Fully mixed concentration $\overline{C}$ = 2 g.m^{-3}
Thus for C = 4 g.m^{-3} C^* = 2
and for C = 8 g.m^{-3} C^* = 4

(i) *Transverse line source*
From Fig. 2.1c
(a) $x_S D_y/Ud^2 = 0.020$ x_S = 10m y_S = 0.24 m
(b) $x_S D_y/Ud^2 = 0.005$ x_S = 2.5m y_S = 0.12 m

(ii) *Plate diffuser*
Approximate concentration contours can be estimated assuming two point sources, each with mass flow rate 10 g.s^{-1}, location at y = 0.45 and y = 0.55 m. Concentrations are found by tracing two sets of contours from Fig. 2.1c as shown in Fig. 2.3a and combining them to give Fig. 2.3b. Then
(a) $x_S D_y/Ud^2 = 0.018$ x_S = 9m y_S = 0.26 m
(b) $x_S D_y/Ud^2 = 0.004$ x_S = 2m y_S = 0.16 m

NOTE: Predictions could be improved slightly by assuming three point sources each with mass flow rate 6.67 g.s^{-1} located at y = 0.433, y = 0.50 and y = 0.567 *m,* and superposing three sets of contours. They could be improved still further assuming four point sources, and so on. In practice there is little point in combining more than five sets of contours.

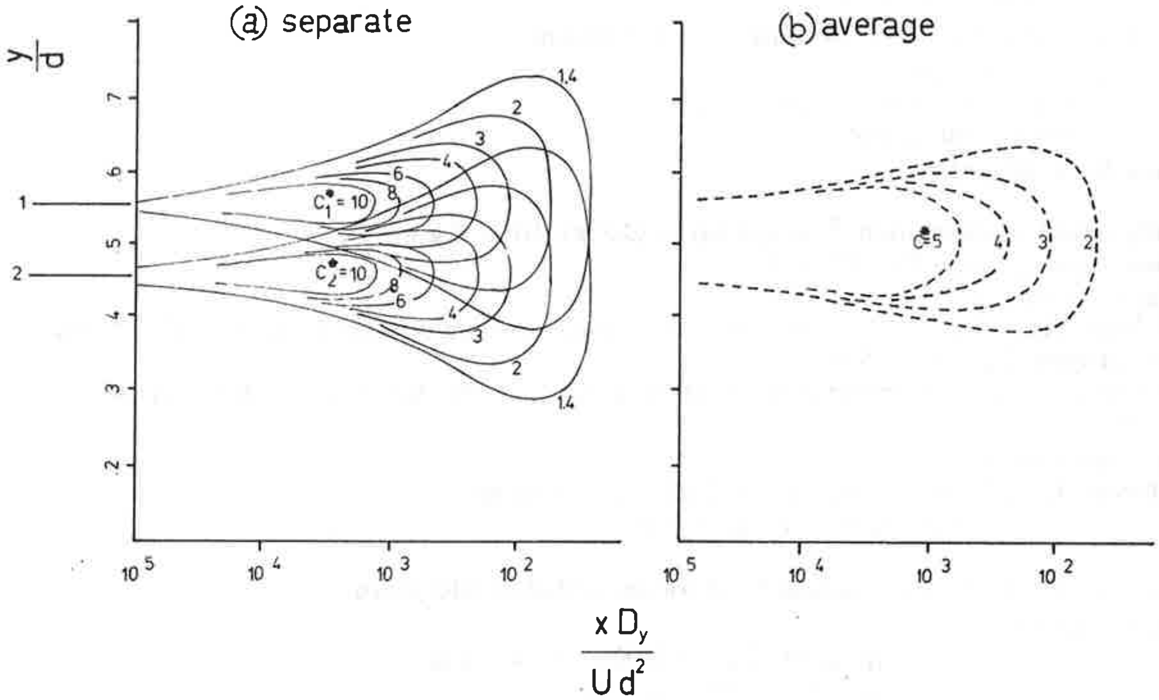


Figure 2.3 Concentration contours downstream from a multi-point source, example 2.6.5.

Example 2.6.6 If the test channel is tidal with velocity and salinity gradients over one tidal cycle estimated from field measurements as shown in Fig. 2.4, estimate how quickly tracer released at mid-depth becomes fully mixed.

From Fig. 2.4 the average values over the tidal cycle are

$$\frac{1}{\rho}\frac{\partial \rho}{\partial y} = 2$$

$$\frac{\partial U}{\partial y} = 0.85$$

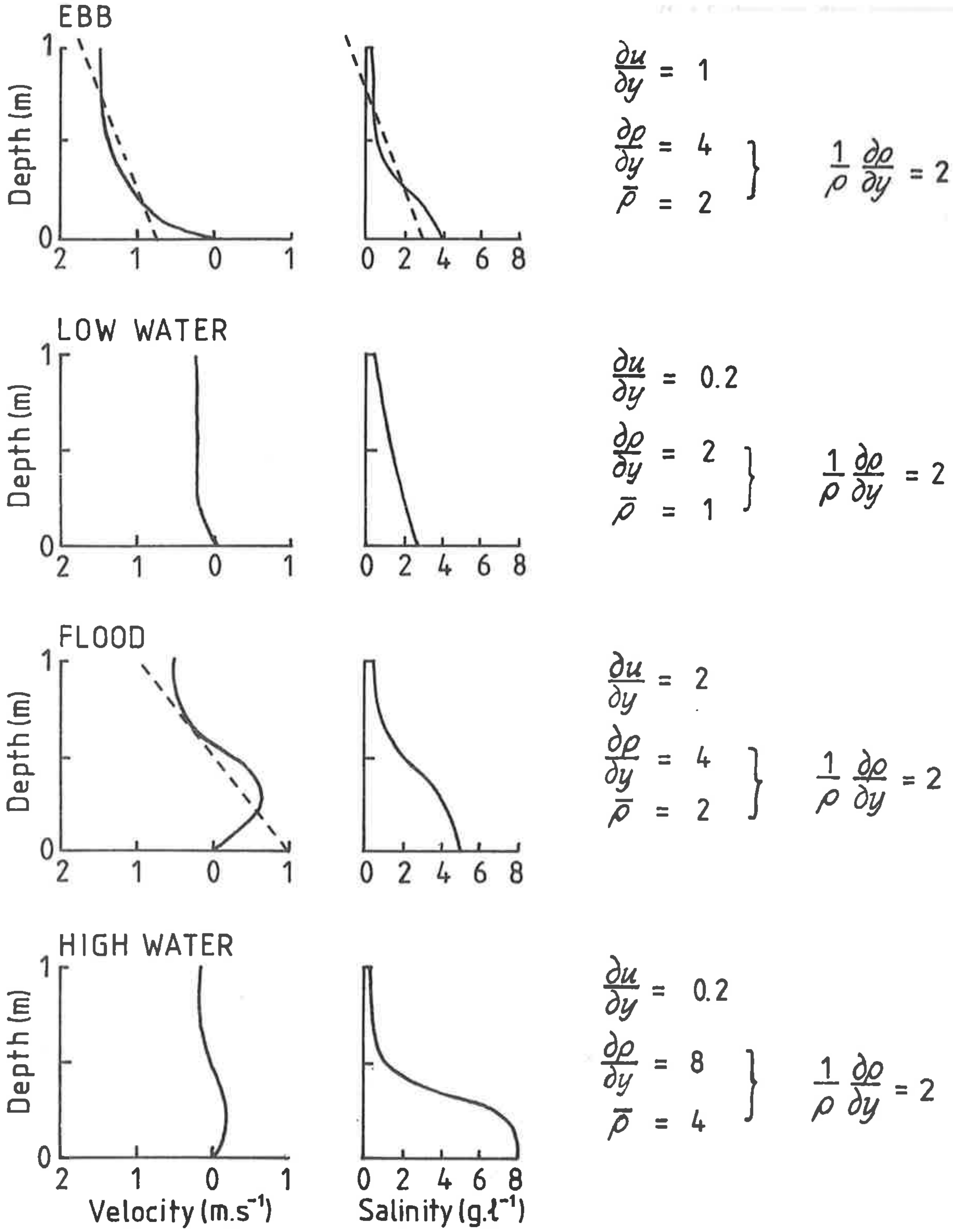


Figure 2.4 Velocity and salinity profiles, example 2.6.6

From equation 2.4

$$Ri = \frac{9.81 \times 2}{(0.85)^2} = 27$$

Then from equation 2.5, using coefficients $a = 0.276 \quad b = -2.00$

$$D_S = \frac{20}{(1 + 0.276 \times 27)^2} = 0.28\ \mathrm{cm^2.s^{-1}}$$

From equation 2.11

$$x_S = 0.1\ Ud/D_S = 3600\ \mathrm{m}$$

(compare with example 2.6.2).

Example 2.6.7 For the channel in example 2.6.3 estimate

(a) total length, and

(b) width of the plume

in which concentration exceeds 200 $\mathrm{g.m^{-3}}$ for an outfall at the river bed with mass flow 20 $\mathrm{g.s^{-1}}$.

$$\overline{C} = q/Udb = 2\ \mathrm{g.m^{-3}} \quad C^* = 100$$

Previously $D_y = 20\ \mathrm{cm}^2.^{-1}$. To account for reduced dispersion near the bed take $D_y = 0.2 - 2\ \mathrm{cm^2.s^{-1}}$.

(a) From Fig. 2.2a, the $C^* = 100$ contour has total length

$$\frac{x_S D_y}{Ud^2} = 0.050$$

Thus for $D_y = 0.2\ \mathrm{cm^2.s^{-1}}$, $x_S = 2500$ m

$D_y = 2.0\ \mathrm{cm^2.s^{-1}}$, $x_S = 250$ m

(Note for $D_y = 20\ \mathrm{cm^2.s^{-1}}$, $x_S = 25$ m)

(b) From Fig. 2.2a, the $C^* = 100$ contour has maximum width

$$\frac{y_S}{d} = 0.25$$

Thus $y_S = 0.25$ m

3.0 TRANSVERSE MIXING

3.1 Mechanisms causing transverse mixing

The rate of transverse mixing in rivers is determined by two processes: turbulent mixing and transverse secondary currents.

When turbulent mixing dominates and the scale of turbulence is controlled by the depth, the dispersion coefficient is proportional to depth and shear velocity (Fischer 1973). Because larger length scales of turbulence are involved (because larger eddies can develop in the transverse direction than in the vertical direction) D_z is greater than D_y by a factor of between 2 and 3 (see Table 2.1 and 3.1). Laboratory studies indicate that for tracers

$$0.08 < D_z/d\,u^* < 0.18 \qquad \text{average } 0.15 \qquad (3.1)$$

where D_z = transverse dispersion coefficient, d = depth and u^* = shear velocity. D_z decreases with depth in much the same manner as mean velocity (Fischer 1973).

In straight natural channels the rate of transverse dispersion is higher than in equation 3.1 because the thalweg tends to meander and hence induce secondary currents. It appears that in such channels (Fischer 1973)

$$0.23 < D_z/d\,u^* < 0.25 \qquad (3.2)$$

Bends and changes in channel cross-section result in stronger transverse circulations which increase transverse mixing rates (Fischer 1973)

$$0.25 < D_z/d\,u^* < 1.6 \qquad (3.3)$$

Table 3.1 Reported transverse dispersion coefficients

Reference (see note)	Channel	d cm	b m	u^* cm.s^{-1}	D_z cm^2.s^{-1}	$\frac{D_z}{du^*}$	$\frac{D_z}{bu^*}$ x10^3
1	straight laboratory channels	1.3–5.0	0.3–0.6	0.90–2.8	0.33–14	0.11–0.26	2.7–16.2
1		9.7	1.2	0.60	0.92	0.16	12.9
1		4.0–6.5	1.2	1.1–2.1	0.86–1.6	0.15–0.18	4.8–8.1
2		15.8	0.69	5.2	6.6	0.08	18.3
2		1.2	0.36	1.6	0.31	0.16	5.3
2		15-37	2.38	3.8-6.0	9.6-36.9	0.17-0.18	11-26
2		7.3–10.2	0.80	0.83–1.21	0.9–1.2	0.11–0.13	12.2–13.8
1		1.5–22	0.85–1.1	1.4–5.1	0.64–7.5	0.09–0.24	3.5–21.4
3		4.1–11.1	1.1	1.9–4.0	1.1–3.6	0.14–0.16	4.8–16.4
4		12–13	0.60	3.0–16.3	3.7–36	0.10–0.18	21–38.3
2	sinuous laboratory channels	–	–	–	–	0.51–2.4	–
2		–	–	–	–	0.62–1.2	–
5		–	–	–	–	0.66–1.7	–
2	straight canal	67–68	18.3	6.1–6.3	102	0.24–0.25	9
6	Waal canal	470	265	5.9	1180-1580	0.43–0.57	7.6–10.2
6	Ijssel canal	400	70	7.8	1600	0.51	29.2
7	Fraser estuary	1040	~1000	2.7–7.0	3200–4800	0.44–1.61	6.9–11.9
5	Cordova estuary	–	–	–	–	0.42	–
5	Gironde estuary	–	–	–	–	1.0	–
5	San Francisco Bay	–	–	–	–	1.0	–
8	Waikato River	280	100	6.0	300–1000	0.18–0.60	5.0–16.7
2	Missouri River	270	200	7.4	1200	0.6**	8.1
2	Columbia River	300	300	8.8	1860	0.70	7.0
5	Delaware River	–	–	–	10000	1.2	–

**10 on sharp bends

1 Lau & Krishnappan (1977)
2 Fischer (1973)
3 Prych (1970)
4 Miller & Richardson (1974)
5 Fischer (1976)
6 Holley & Abraham (1973)
7 Ward (1976)
8 Rutherford *et al.* (1980)

Laboratory data in sinuous channels fit (Fischer 1973)

$$D_z \cong 0.25 \frac{U^2 d^3}{K^5 R^2 u^*} \tag{3.4}$$

where R = radius of curvature, and K = von Kármán's constant (= 0.4). In very rough channels, secondary currents may be destroyed and D_z reduced. Table 3.1 summarises a number of published data.

In both laboratory and natural channels, $D_z/d\,u^*$ increases with aspect ratio b/d (Fischer 1973). This indicates that transverse dispersion is affected by secondary currents whose importance depends on channel width. In rivers the transverse dispersion coefficient may, therefore, be more closely related to width and shear velocity than to depth and shear velocity (Lau & Krishnappan 1977). Figure 3.1 summarises published data on this basis.

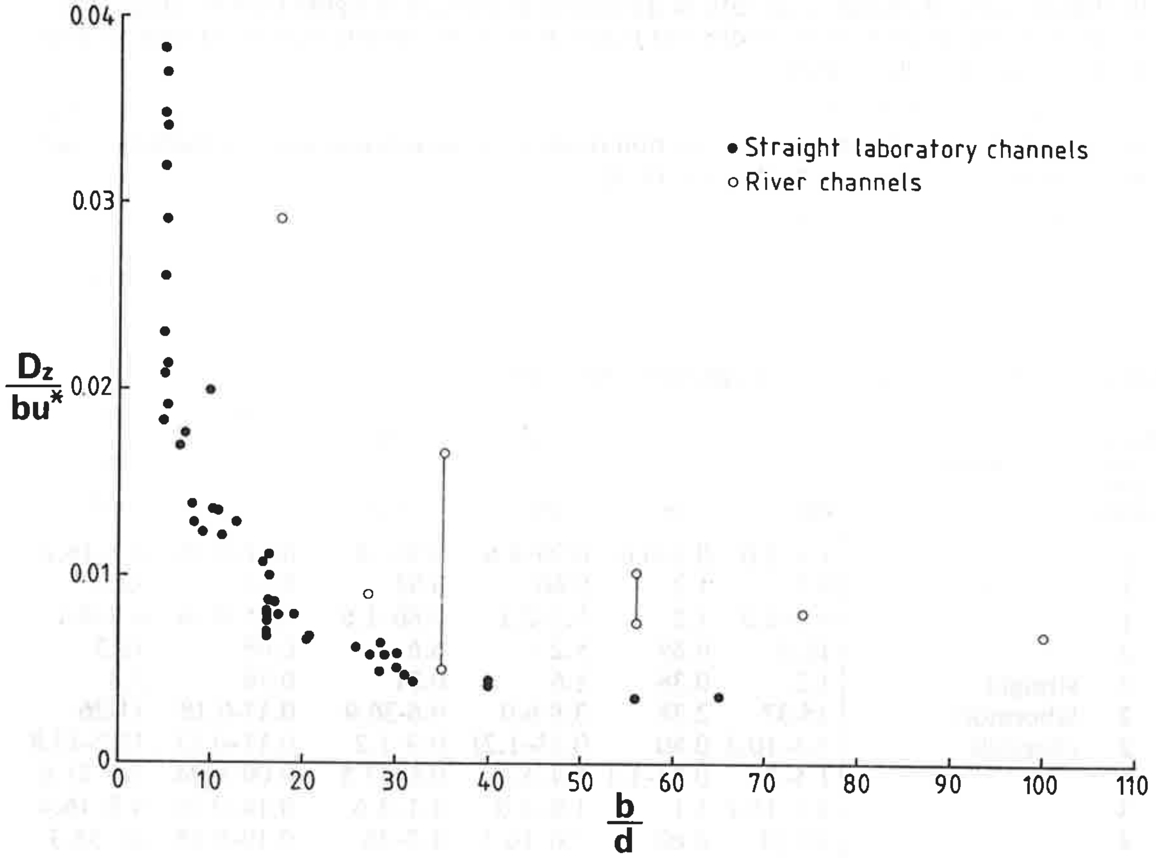


Figure 3.1 Reported transverse dispersion coefficients (references to published data are given in Table 3.1).

3.2 Effects of density stratification

As discussed in Section 2 the effect of density stratification, such as may be encountered in a tidal channel, is to suppress vertical mixing. Meteorological observations indicate that stratification in the atmosphere reduces transverse dispersion as well. In non-uniform stratified flow, however, tidal changes in level may induce transverse density currents which increase transverse dispersion (Fischer 1976). Thus, at present, it is not possible to quantify the effects of stratification on lateral dispersion and it must be neglected.

3.3 Effects of non-neutrally buoyant effluents

When a non-neutrally buoyant effluent is discharged into a channel it either sinks or rises, and in so doing, induces transverse secondary circulations. These secondary circulations increase the rate of transverse mixing in the immediate vicinity of the outfall. Gradually, however, mixing spreads the effluent uniformly over the depth, the driving force of the secondary circulation diminishes, and the transverse dispersion coefficient approaches that for a non-neutrally buoyant effluent.

This process has been quantified in laboratory channels (Prych 1970) and it appears that the increase in spreading rate is greater for a buoyant than for a heavy effluent. Transverse dispersion coefficients 2–4 times that for a neutrally buoyant effluent were observed within a distance $0 < x < 10\text{–}20b$ of the outfall.

3.4 Transverse mixing downstream from a point source

An important practical problem is to predict how quickly a tracer mixes transversely downstream from an outfall. Although very close to the outfall both vertical and transverse dispersion occur and the problem is two-dimensional, in most rivers tracer quickly becomes vertically well-mixed. Thus transverse mixing in rivers can normally be considered as quasi one-dimensional (see Table 1.1). In this simplified analysis velocity and dispersion coefficient are assumed uniform across the channel as an approximation to turbulent flow.

Figure 3.2 shows concentrations in the horizontal plane (x–z) downstream from steady point sources located at three points across the channel. The non-dimensional variables used in this figure are

$$C^* = C/\overline{C} = CUbd/q \qquad (3.5)$$

$$z^* = z/b \qquad (3.6)$$

$$x^* = x\,D_z/Ub^2 \qquad (3.7)$$

See equations 2.6 to 2.8 for a description of these variables.

As discussed in Section 2.3, the rate of dispersion is low close to a boundary. Thus Fig. 3.2a should only be used to give a preliminary estimate of concentrations and a very conservative value of D_z should be employed (say 1–10% of the cross-section mean value). This preliminary estimate should be checked using field tests and more sophisticated modelling.

Figures 2.1 and 3.2 are identical except for slightly different non-dimensional variables (see equations 2.6 to 2.8). By analogy with equations 2.11 and 2.12 effluent becomes well-mixed across the channel within a distance

$$x_m \cong 0.1\;Ub^2/D_z \qquad (3.8)$$

of an outfall in mid-channel (Shen 1973) and within

$$x_m \cong 0.4\;Ub^2/D_z \qquad (3.9)$$

of an outfall at either bank (Shen 1973). As discussed above the rate of dispersion is low close to a boundary and a conservative estimate of D_z should be used in equation 3.9.

Figure 3.2 can be used to determine the length and width of a plume in which concentrations are above a specified level.

3.5 Worked examples

Consider the same channel as in Section 2.6, viz depth = 1 m, width = 10 m, mean velocity = 1 m.s^{-1}, shear velocity = 3.1 cm.s^{-1}.

Example 3.5.1 Select transverse dispersion coefficients assuming the channel is

(i) straight but rough
(ii) a fairly straight natural river channel
(iii) sinuous with radius of curvature 100 m

(i) From equation 3.1 $D_z = 0.15\,du^* = 50$ cm^2.s^{-1}
Also from Fig. 3.1 for $b/d = 10$,
$D_z/bu^* = 0.012 - 0.020$ for straight smooth channels
and thus $D_z = 37\text{–}62$ cm^2.s^{-1}

(ii) From equation 3.2 $D_z = 0.24\,du^* = 75$ cm^2.s^{-1}

(iii) From equation 3.3 $D_z = 0.25 - 1.6\,du^* = 75\text{–}500$ cm^2.s^{-1}
Also from equation 3.4 $D_z = 800$ cm^2.s^{-1}

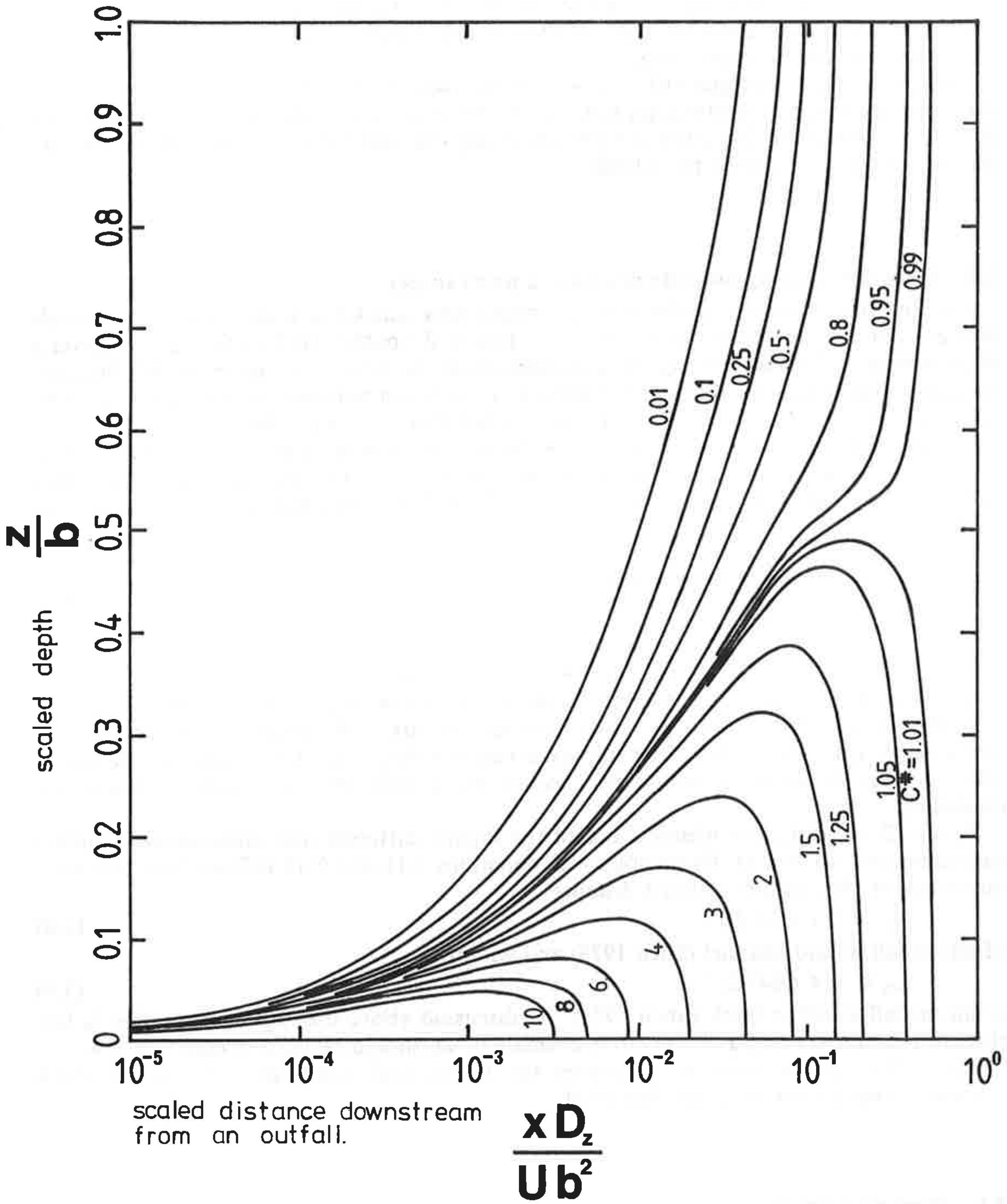


Figure 3.2a Concentration contours downstream from a steady vertical line source located on the bank. (See text for caveat on use of this figure.)

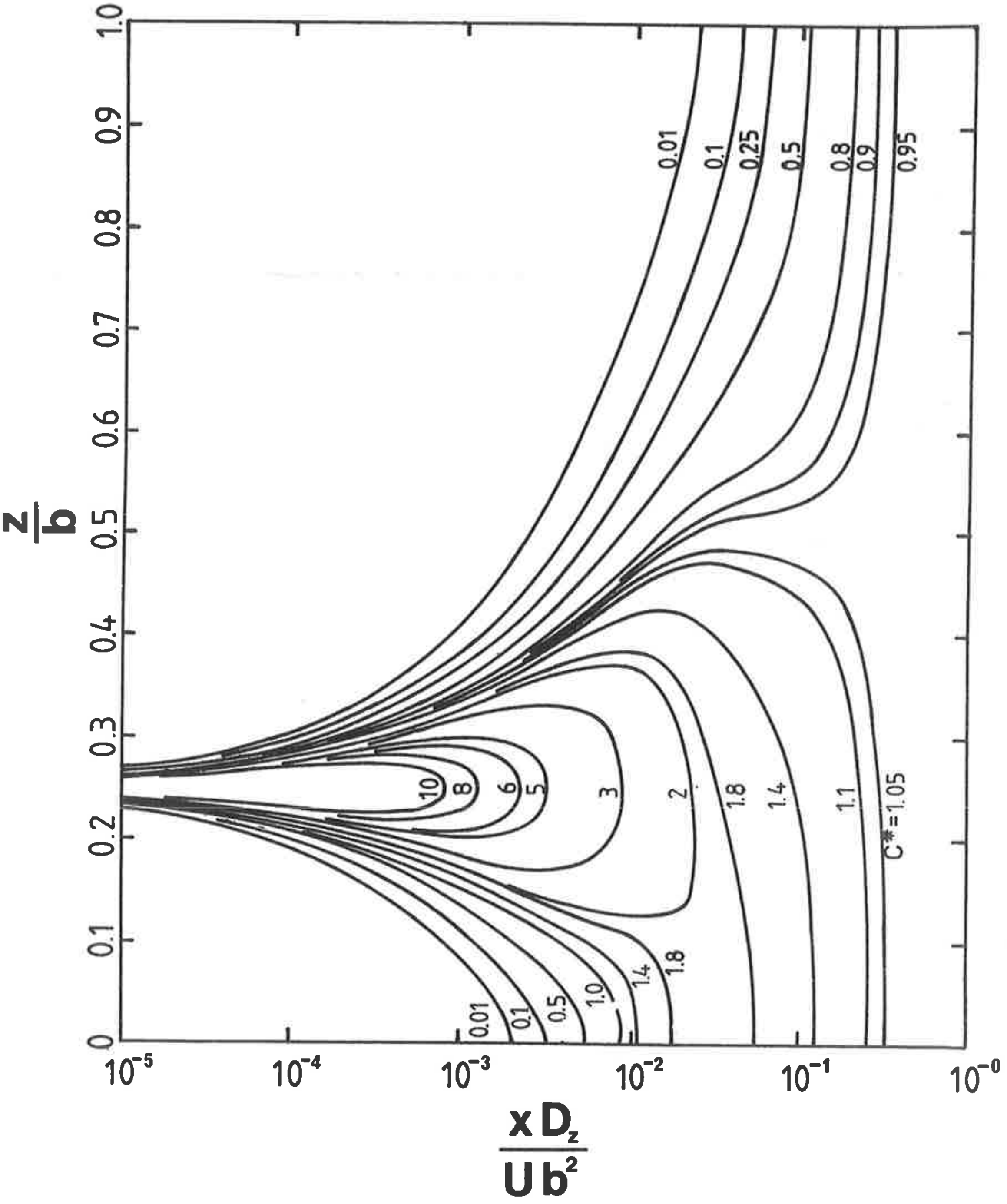


Figure 3.2b Concentration contours downstream from a steady vertical line source located at three-quarters of the width.

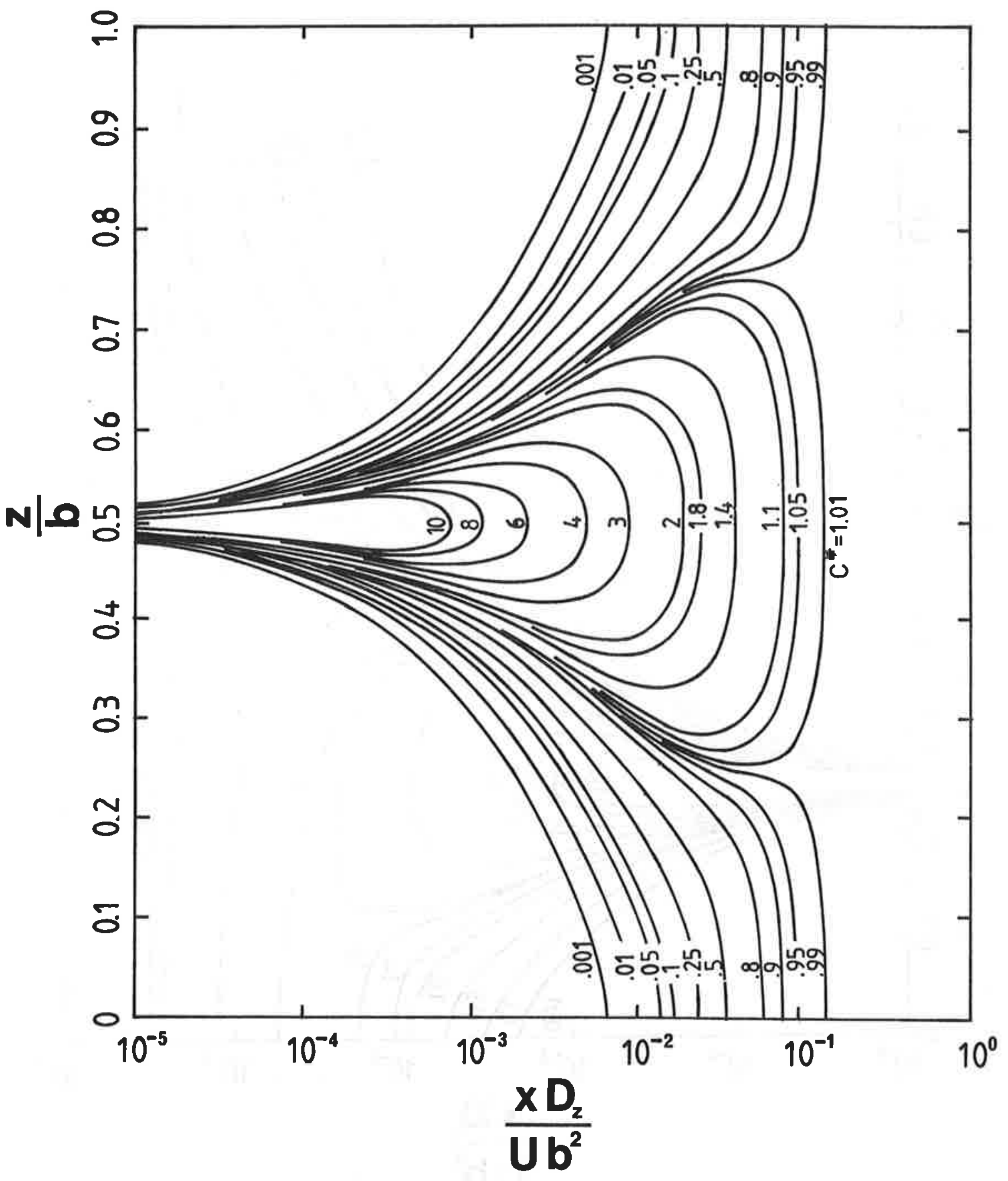


Figure 3.2c Concentration contours downstream from a steady vertical line source located at mid channel.

Example 3.5.2 Estimate how far downstream from an outfall

(a) at either bank, and

(b) in mid-channel

a tracer becomes well-mixed transversely. Take $D_z = 200\ \text{cm}^2.\text{s}^{-1}$.

(a) To account for reduced mixing near the bank take $D_z = 2\text{–}20\ \text{cm}^2.\text{s}^{-1}$. Then from equation 3.9

$x_m = 0.4\ Ub^2/D_z$

For $D_z = 2$ $x_m = 200$ km

For $D_z = 20$ $x_m = 20$ km

(b) Take $D_z = 200\ \text{cm}^2.\text{s}^{-1}$ in this case, then from equation 3.8

$x_m = 0.1\ Ub^2/D_z = 0.5$ km

Example 3.5.3 For an outfall located 2.5 m from one bank, with mass flow rate of $20\ \text{g.s}^{-1}$, estimate

(a) how far downstream concentrations exceed $5\ \text{g.m}^{-3}$, and

(b) over how much of the channel such concentrations spread.

Take $D_z = 200\ \text{cm}^2.\text{s}^{-1}$.

Fully mixed concentration $\overline{C} = q/Udb = 20/1\text{x}1\text{x}10 = 2\ \text{g.m}^{-3}$

Thus for $C = 5\ \text{g.m}^{-3}$ $C^* = 2.5$, from equation 3.5

From Fig. 3.2b

(a) $x_s D_z/Ub^2 = 0.012$ $x_s = 60$ m

(b) $z_s/b = 0.20$ $z_s = 2$ m

Example 3.5.4 Repeat example 3.5.3 for a buoyant effluent released from the river bed. From Section 3.3, take D_z three times higher than previously within $x = 100$ m of the outfall.

From Figure 3.3b

(a) $x_s D_z/Ub^2 = 0.012$ $x_s = 20$ m

(b) as before $z_s/b = 0.20$ $z_s = 2$ m

Example 3.5.5 For an outfall with mass flow rate $10\ \text{g.s}^{-1}$, compare the length, x_s, and z_s, of plumes in which concentrations exceed

(i) $2\ \text{g.m}^{-3}$

(ii) $4\ \text{g.m}^{-3}$

in the case of

(a) a point source located at mid-depth

(b) a diffuser pipe between $z = 4$ and $z = 6$ m from the bank.

Take $D_z = 800\ \text{cm}^2.\text{s}^{-1}$

Fully mixed concentration $\overline{C} = 10/1\text{x}1\text{x}10 = 1\ \text{g.m}^{-3}$

From equation 3.5

(i) for $C = 2\ \text{g.m}^{-3}$ $C^* = 2$

(ii) for $C = 4\ \text{g.m}^{-3}$ $C^* = 4$

(a) From Fig. 3.2c

(i) $x_s D_z/Ub^2 = 0.020 \rightarrow x_s = 25$ m $z_s/b = 0.24 \rightarrow z_s = 2.4$ m

(ii) $x_s D_z/Ub^2 = 0.005 \rightarrow x_s = 6.3$ m $z_s/b = 0.12 \rightarrow z_s = 1.2$ m

(b) The problem is similar to example 2.5, Section 2.6. Figure 2.3 can be used to obtain the solution.

(i) $x_s D_z/Ub^2 = 0.018 \rightarrow x_s = 23$ m $z_s/b = 0.26 \rightarrow z_s = 2.6$ m

(ii) $x_s D_z/Ub^2 = 0.004 \rightarrow x_s = 5$ m $z_s/b = 0.16 \rightarrow z_s = 1.6$ m

4.0 LONGITUDINAL DISPERSION

4.1 Mechanisms causing longitudinal dispersion

Longitudinal dispersion is the spreading of tracer along the axis of flow. It results in the attenuation of peak concentrations, as illustrated in Fig. 4.1. Longitudinal dispersion is largely the result of non-uniformities of velocity in the channel cross-section (Fischer 1973). These transport material downstream faster in the main stream than near the banks and bed, thereby causing the cross-section mean concentration to spread longitudinally (see Fig. 4.2). Vertical and transverse dispersion counteract the effects of velocity gradients.

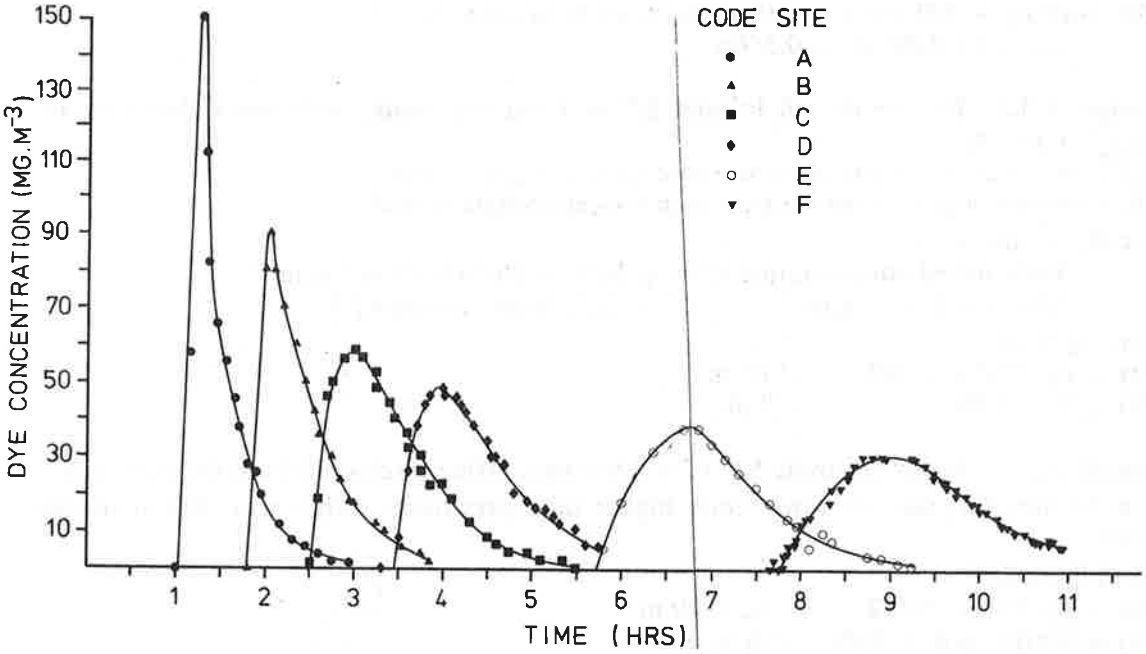


Figure 4.1 Longitudinal dispersion of dye in the Waikato River (after Rutherford *et al.* 1980).

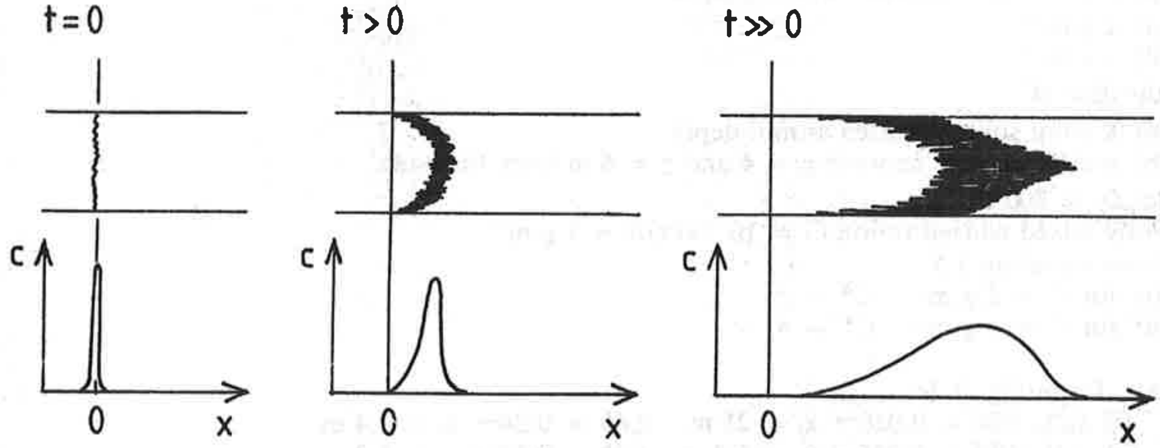


Figure 4.2 The effect of transverse velocity gradients and dispersion on longitudinal dispersion.

4.2 Mathematical model for longitudinal dispersion

In the immediate vicinity of an outfall, longitudinal dispersion must be described by complex models in two or three dimensions. Some way downstream, however, a simpler one-dimensional Fickian model can be used

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = D_x \frac{\partial^2 C}{\partial x^2} \tag{4.1}$$

where D_x = longitudinal dispersion coefficient. Equation 4.1 does not apply within the

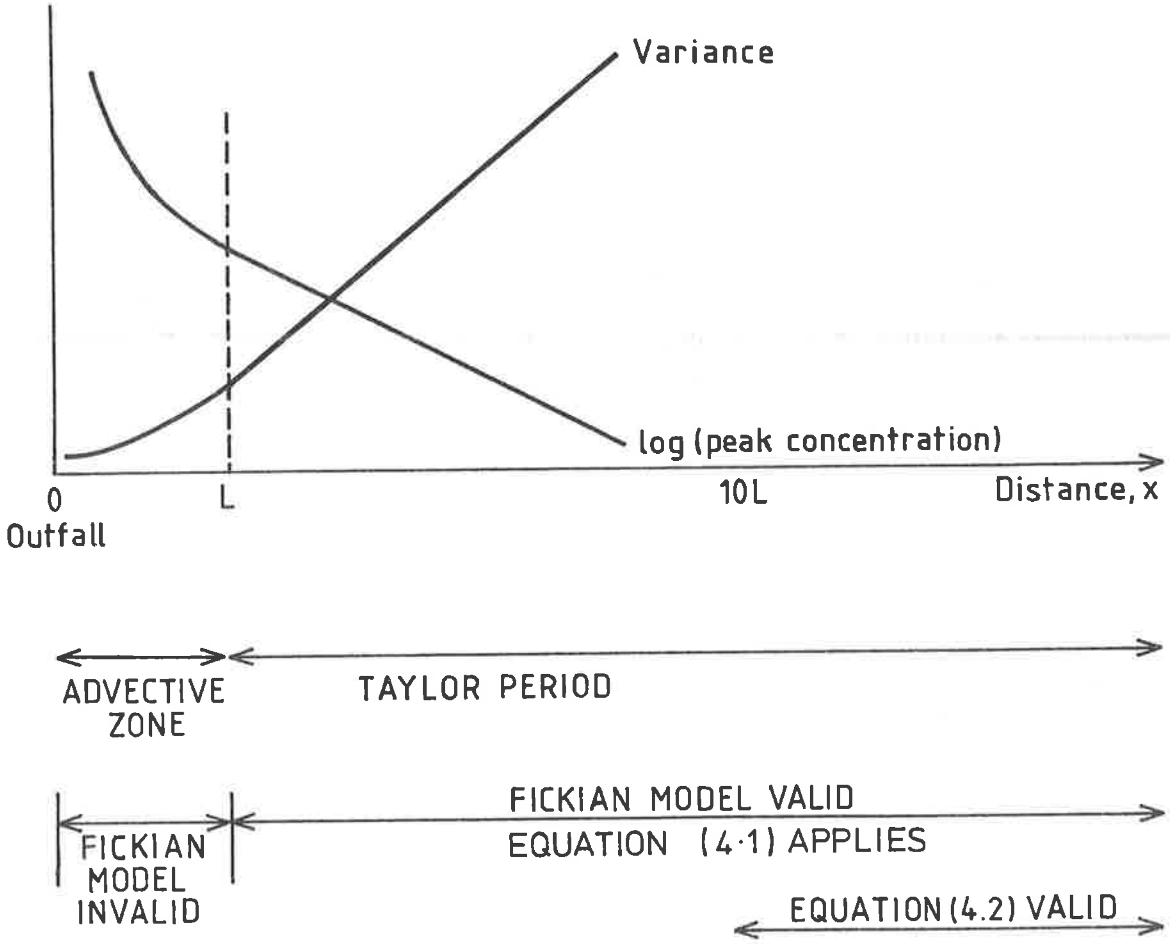


Figure 4.3 How variance and peak concentration change with distance below a point discharge.

"advective zone" (see Fig. 4.3). The length of this zone (Fischer 1973) is

$$L \cong k\, b^2\, U/R\, u^* \qquad (4.2)$$

where k = constant with values given in Table 4.1, b = channel width, U = mean velocity, R = hydraulic radius and u^* = shear velocity.

A wide range of values of D_x has been measured in rivers and laboratory channels. Part of this variation may be attributable to measurements being made in the "advective zone" where D_x is not constant, but much of the variation reflects real differences between channels resulting from differences in channel geometry, turbulent diffusion rates and dead-zones.

Table 4.1 Length of the advective zone for various types of channel, $k = LRu^*/b^2U$

	uniform smooth⁽¹⁾	non-uniform smooth⁽¹⁾	non-uniform large "dead-zones"⁽²⁾
point source mid-channel	0.5⁽³⁾–1.1⁽⁴⁾	1–4⁽⁵⁾	
point source near bank	1.0⁽³⁾–2.5⁽⁴⁾	5–15⁽⁵⁾	135–340⁽⁵⁾

NOTES:
(1) no "dead-zones"
(2) "dead-zones" are zones where the water is nearly stagnant, e.g., under rocks, in holes, behind obstacles etc.
(3) Fischer (1973)
(4) Chatwin (1972)
(5) extrapolated from line source data of Valentine (1978)

4.3 Longitudinal mixing below an instantaneous point source

For an instantaneous point injection into a uniform channel, concentrations can be predicted (Fischer 1973) from

$$C(x, t) = \frac{W}{A\sqrt{4\pi D_x t}} \exp\left(-\frac{(x-Ut)^2}{4D_x t}\right) \tag{4.3}$$

where A = channel cross-sectional area, and W = mass discharged at $x = 0$, $t = 0$. Strictly this solution is valid only when $x >> 10L$ where L = length of the advective zone (see equation 4.2), because it neglects the effects of the advective zone. However, it provides an approximate description of concentration even moderately close to the outfall, although D_x will vary with distance from the outfall (normally decreasing) and the shape of the profiles may be considerably less symmetrical than predicted.

At a specified time after release, t, equation 4.3 indicates that the concentration versus distance profile is bell-shaped and symmetrical.

The peak concentration is

$$C_p = \frac{W}{A\sqrt{4\pi D_x t}} \tag{4.4}$$

and occurs at

$$x_p = Ut \tag{4.5}$$

Concentrations exceed C_p/a, where $a>1$, over a distance

$$x_s = 4\sqrt{D_x t \ln(a)} \tag{4.6}$$

At a specified distance from the outfall, x, however, the concentration versus time profile is not symmetrical. The peak concentration occurs at

$$t_p = -\frac{D_x}{U^2} + \sqrt{\frac{D_x^2}{U^4} + \frac{x^2}{U^2}} \tag{4.7}$$

slightly earlier than implied by equation 4.5. The peak concentration can be calculated by substituting the result of equation 4.7 into equation 4.3.

Figure 4.4 shows the times at which various concentrations occur at specified locations. The non-dimensional variables used are

$$C^* = \frac{CA}{W} \frac{D_x}{U} \tag{4.8}$$

$$x^* = \frac{xU}{D_x} \tag{4.9}$$

$$t^* = \frac{tU^2}{D_x} \tag{4.10}$$

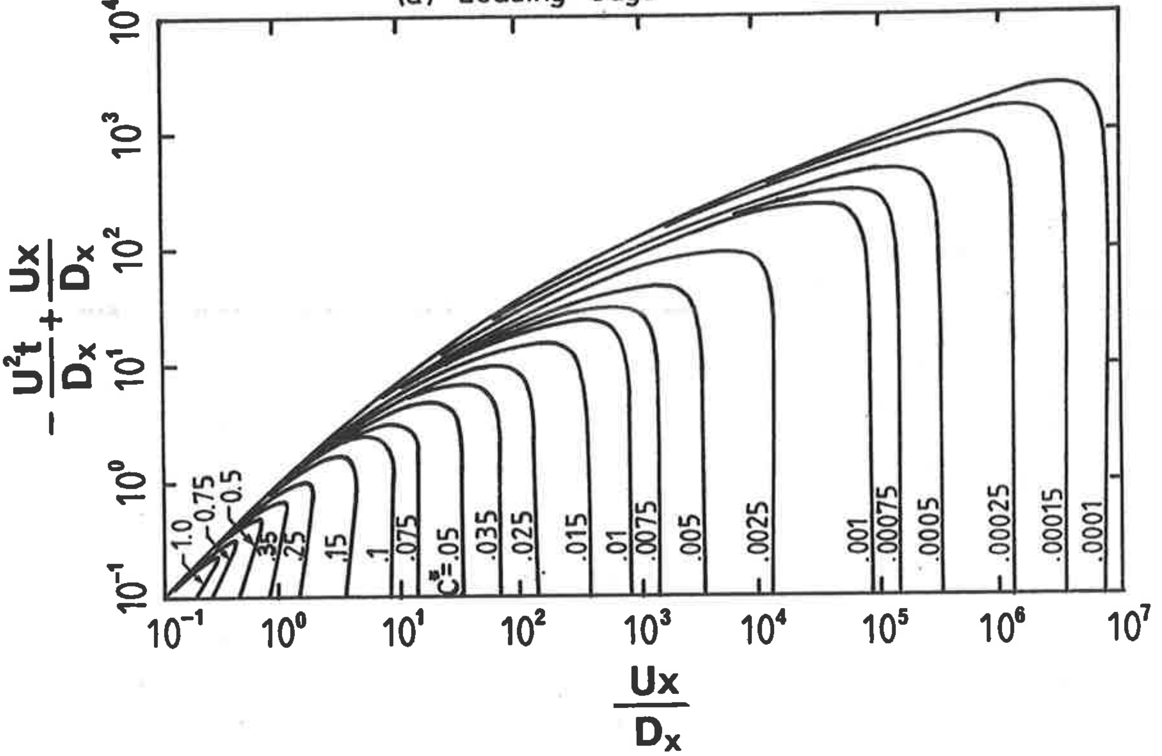
Figure 4.4a indicates how much earlier than the mean travel time, x/U, concentrations first reach C^* while Fig. 4.4b indicates how much later than the mean travel time concentrations drop below C^*.

By comparing the lengths of vertical lines drawn through a particular value of x^* to a given C^* contour in Fig. 4.4a and 4.4b it can be seen that concentration versus time profiles are markedly asymmetrical. The degree of asymmetry decreases as x^* increases but never entirely vanishes.

The peak concentration at a specified site, x^*, can be deduced from the largest value of C^* on a vertical line passing through x^*.

4.4 Longitudinal mixing below a time-varying point source

For preliminary analysis of a problem in longitudinal dispersion, knowledge of the fully-mixed concentration and the behaviour of an instantaneous point discharge is often sufficient.


(a) Leading edge

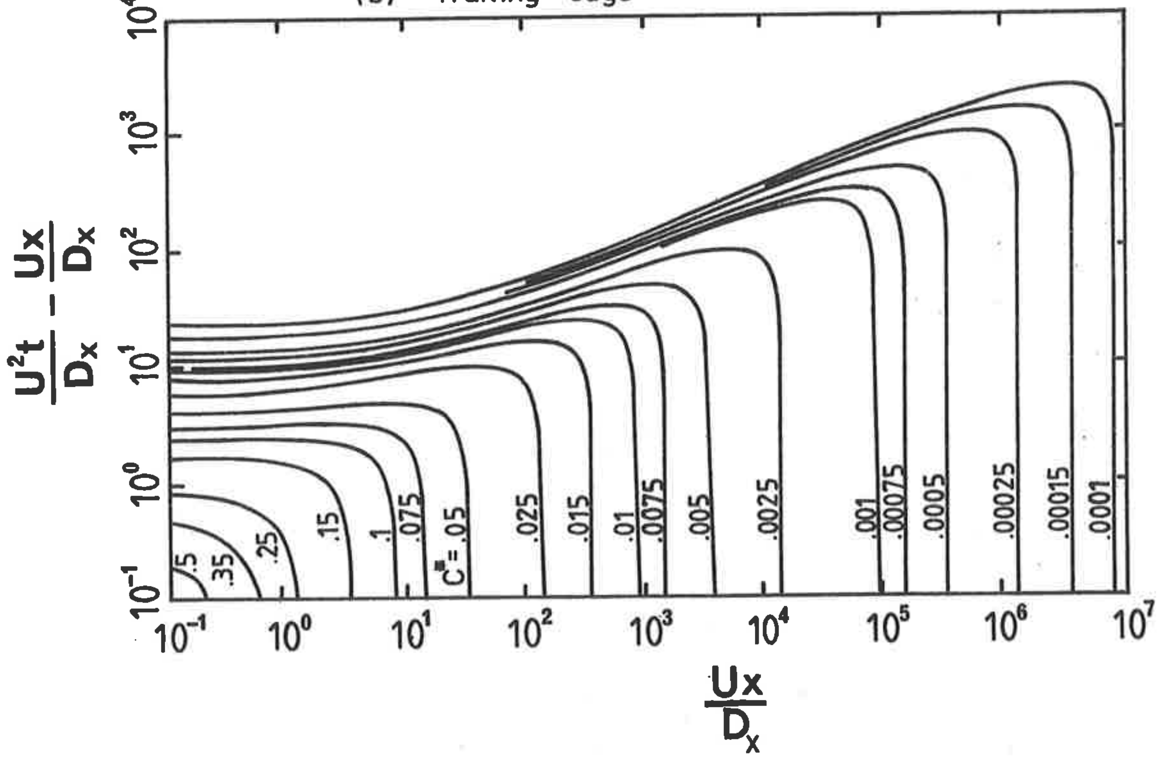

(b) Trailing edge

Figure 4.4 Concentration contours downstream from an instantaneous point discharge.

Table 4.2 Summary of reported longitudinal dispersion coefficients

(a) FIELD STUDIES

Reference	Channel	depth	width	shear velocity	discharge	dispersion coefficient	
		d cm	b m	u^* cm.s^{-1}	Q m^3.s^{-1}	D_x m^2.s^{-1}	D_x/du^*
1	Yuma Mesa A Canal	345	–	3.45	–	0.76	8.6
1	Chicago Ship Canal	807	48.8	1.91	–	3.0	20
1	River Derwent	25	–	14.0	–	4.6	131
2	Monocacy River	32.2	35	4.35	2.4	4.7	332
		44.5	36.5	5.12	5.2	13.9	610
		87.6	47.6	7.18	18.4	37.2	591
1	Green-Duwamish Rv	110	20	4.9	–	6.5–8.5	120–160
2	Concite River	25.5	12.5	4.42	1.0	7.0	620
		41.2	15.9	5.61	2.4	13.9	600
1,2	Clinch River	58	36	4.9	6.8	8.1	280
		84	47	6.7	9.2	14	235
		210	53	10.7	51	47	210
		210	60	10.4	85	54	245
2	Antietam Creek	28.7	15.9	6.16	2.0	9.3	390
		51.6	19.8	7.11	4.4	16.3	440
		70.6	24.4	8.32	8.9	25.6	435
2	Elkhorn River	30.1	33	4.64	4.3	9.3	666
		42.0	50.9	4.68	10.0	20.9	1063
1	Powell River	85	34	5.5	4.0	9.5	200
1,2	Copper Creek	49	16	8.0	1.5	9.5	245
		40	19	11.6	13.7	9.9	220
		49	16	8.0	1.5	20	500
		85	18	10.0	8.5	21	250
1,2	Coachella Canal	156	24	4.3	26.9	9.6	140
3	Lucas Creek	–	–	–	–	10	–
10	Fraser Estuary	–	–	2.7–7.0	–	10.7–12.7	18–38
2	Bayou Anacoco	41.5	19.8	4.51	2.4	13.9	743
		93.7	25.9	6.78	8.2	32.5	511
		92.1	36.6	6.72	13.5	39.5	638
2	Muddy Creek	80.8	13.4	8.11	4.0	13.9	212
		120	19.5	9.88	10.6	32.5	274
2	John Day	56	24	14.0	14.2	13.9	177
		246	34	18.1	69	65	146
1	Sacramento River	400	–	5.1	–	15	74
1	South Platte River	46	–	6.9	–	16.2	510
2	Amite River	80.7	36.6	6.95	8.6	23.2	414
		80.1	42.4	6.92	14.2	30.2	545
4	Manawatu River	100	25	10	26	26–45	260–450
2	White River	54.7	67	4.40	12.8	30.2	1255
2	Chattahoochee Rv	113	65.5	7.58	30	32.5	379
5	Waikato River	200–300	70–130	5.4–5.8	160	33–70	240–510
	average	250	100	5.5	160	50	360
2	Nooksack River	76	64	–	33	34.9	–
		293	86	53.1	300	153.4	98.6
2	Sabine River, Texas	98.5	35	4.17	7.4	39.5	961.7
2	Sabine River	204	104	5.47	119	316	2832
		475	127	8.36	389	670	1687
2	Wind/Bighorn Rv	97.7	67	11.2	58	41.9	383
		216.5	68.6	16.6	231	163	454
2	Susquehanna River	135	203	6.51	106	92.9	1057
2	Yadkin	233	70	10.0	71	112	481
		385	71.6	12.9	213	260	524
9	Mississippi	–	–	–	10,310	185–232	–
		–	–	–	22,600	650–700	900
1,2	Missouri River	223	183	6.61	380	465	3155
		270	200	7.4	–	1500	7500
		356	201	8.36	913	837	2812
		311	197	7.81	935	892	3672

(b) LABORATORY STUDIES

Ref-erence	Channel	depth d cm	width b cm	shear velocity u^* cm.s^{-1}	mean velocity U cm.s^{-1}	dispersion coefficient D_x m^{2}s^{-1} $\times 10^2$	D_x/du^*	D_x/Ru^*
1	rough sides	2.1–4.7	20–43	2.0–3.9	25–48	12–42	150-390	190–640
6	rough bed	1.0–7.0	58.4	2.0–10.0	16–50	0.9–4.0	11–42	-
7	rough bed	3.8–15	61	1.8–4.0	12–43	1.0–8.3	14–60	-
8	rough bed	13	59.7	3.0–16	30-81	5-610	14–287	9.6–200

1 Fischer (1973)
2 McQuivey & Keefer (1974)
3 Harris (pers. comm.)
4 Rutherford (1979)
5 Rutherford *et al.* (1980)
6 Valentine (1978)
7 El-Hadi & Davar (1976)
8 Miller & Richardson (1974)
9 McQuivey & Keefer (1976)
10 Ward (1976)

For more detailed analysis, however, knowledge of how concentrations change with time below an unsteady point source may be required. These can be predicted by superposing solutions derived from equations 4.3.

This is straightforward, if somewhat tedious, to do manually. See Section 4.5 for a worked example.

4.5 Worked examples

Consider the same channel as in Section 2.6.

Example 4.5.1 Select a likely value of D_x.

No channel in Table 4.2 matches exactly the depth, width and shear velocity of the channel in this problem, but the Concite River and Muddy Creek, are similar. Thus D_x could be expected to fall in the range 275–620 m^2.s^{-1}. Assume an average value of 500 m^2.s^{-1}.

Example 4.5.2 Estimate the length of the advective zone for point sources located

(a) in mid-stream, and
(b) near either bank

Assuming the channel is non-uniform and fairly smooth, from equation 4.2 and Table 4.1

$$\text{(a)}\ L = 1\text{-}4\ \frac{b^2U}{Ru^*} = 3.8 - 15.3\ \text{km}$$

$$\text{(b)}\ L = 5\text{-}15\ \frac{b^2U}{Ru^*} = 19 - 58\ \text{km}$$

Assuming the channel is uniform and smooth, from equation 4.2 and Table 4.1

$$\text{(a)}\ L = 0.5 - 1.1\ \frac{b^2U}{Ru^*} \doteq 1.9 - 4.2\ \text{km}$$

$$\text{(b)}\ L = 1.0 - 2.5\ \frac{b^2U}{Ru^*} = 3.8 - 9.6\ \text{km}$$

Example 4.5.3 Given $W = 1$ kg, calculate the distance below the outfall where the peak concentration drops below

(a) 0.1 g.m^{-3} and

(b) 10 mg.m^{-3}

(a) From equation 4.8

$$C^* = \frac{CA}{W}\frac{D_x}{U} = 0.50$$

From Fig. 4.4 the $C^* = 0.50$ contour reaches $x^* = 0.63$

Thus from equation 4.9 x = 915 m

NOTE: this is well within the advective zone calculated above and may be inaccurate.

(b) $C^* = 0.05$ $x^* = 28$ x = 14 km

Example 4.5.4 Given $W = 1$ kg and $x = 10$ km, calculate when the concentration

(a) first reaches 10 mg.m^{-3} and

(b) drops below 10 mg.m^{-3}

From equation 4.8 $C^* = 0.05$

From equation 4.9 $x^* = 20$

(a) From Fig. 4.4a $x^* - t^* = 3.8$

Thus $t^* = 20 - 3.8$ $t = 8100$ s = 2.25 hours

(b) From Fig. 4.4b $t^* - x^* = 4.7$

Thus $t^* = 20 + 4.7$ $t = 12{,}350$ s = 3.43 hours

Check by substituting in equation 4.3:

For $t = 8100$ s $C = 11.2$ mg.m^{-3}.

For $t = 12{,}350$ s $C = 9.1$ mg.m^{-3}.

Both are close to 10 mg.m^{-3}.

Example 4.5.5 Given the discharge pattern shown in Fig. 4.5 determine

(a) the peak concentration and

(b) the total time during which concentration exceeds 10 mg.m^{-3} at a site 10 km below the outfall.

Approximate the discharge pattern by four instantaneous point discharges of 1, 0.5, 0.5 and 1 kg at 3½, 4½, 5½ and 6½ hours respectively.

Then equation 4.3 can be used to compute the concentration profile at $x = 10$ km for each instantaneous point discharge separately giving:

t	1st slug	2nd slug	3rd slug	4th slug	total
3½	0.00	0.00	0.00	0.00	0.00
4½	0.07	0.00	0.00	0.00	0.07
5½	8.63	0.04	0.00	0.00	8.67
6½	11.79	4.31	0.04	0.00	16.14
7½	5.37	5.89	4.31	0.07	15.64
8½	1.59	2.68	5.89	8.63	18.79
9½	0.38	0.79	2.68	11.79	15.64
10½	0.08	0.19	0.79	5.37	6.43
11½	0.02	0.04	0.19	1.59	1.84
12½	0.00	0.01	0.04	0.38	0.43
13½	0.00	0.00	0.01	0.08	0.09
14½	0.00	0.00	0.00	0.02	0.02
15½	0.00	0.00	0.00	0.00	0.00

Thus (a) ≅ 20 mg.m^{-3}

(b) ≅ 4½ hours

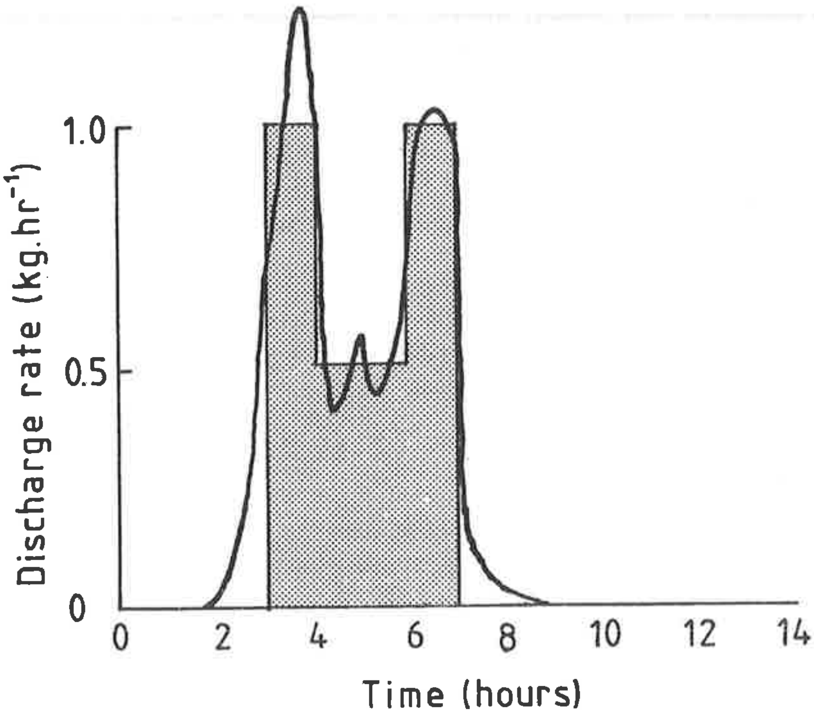


Figure 4.5 Variations of discharge rate with time, example 4.5.5.

5.0 FIELD MEASUREMENTS OF MIXING

5.1 Introduction

If after using the methods described earlier it is considered necessary to undertake a more detailed investigation of a particular mixing problem, then field measurements will be required.

Mixing rates can be measured in three ways:

1. by observing how tracers, either naturally occurring or especially injected, spread out below an outfall;
2. by observing how several surface or submerged drogues spread out;
3. by measuring the velocity distribution and turbulent diffusion rates and inferring dispersion rates.

Methods 1 and 3 are the most popular methods in rivers. Method 2 is used extensively in estuarine and marine studies but is not discussed here.

Two approaches to field studies may be adopted:

1. to measure directly the parameters of interest (e.g., the distance below the outfall where complete mixing is attained);
2. to measure velocities and dispersion coefficients in one part of the river and then to extrapolate (using a mathematical model) to other parts of the river.

The second approach is often preferred because it usually requires less field work and the results can be applied to several problems.

5.2 Channel parameters

Channel parameters such as mean depth, average width, hydraulic radius and bed slope are required for estimating mixing rates and are assumed known from survey data.

5.3 Mean velocity

In investigations of vertical, transverse or longitudinal mixing, estimates of mean velocity, U, are employed.

Point estimates of U can be obtained from gauging data and in a fairly uniform artificial channel, such as a canal, the average from gaugings at several points may be adequate.

In a non-uniform channel mean velocity may be estimated by injecting a slug of tracer and measuring cross-sectional average tracer concentration versus time profiles at several sites. The mean velocity is

$$U = \frac{x_2 - x_1}{t_2 - t_1} \tag{5.1}$$

where U = mean velocity between sites 1 and 2, x_1, x_2 = locations, and t_1, t_2, = times when the centroids of the tracer profiles occur at sites 1 and 2 respectively. Some way below the point of injection of tracer, t_1 and t_2 are closely approximated by the times when the peak concentration occurs (Rutherford *et al.* 1980).

Mean velocity may also be estimated from the time taken for concentration to reach a steady plateau after starting a steady tracer injection.

5.4 Vertical and transverse mixing

5.4.1 *Field techniques* Vertical and transverse dispersion coefficients, D_y and D_z, can be estimated by injecting tracer at a steady rate from a point source (e.g., located at the proposed outfall) and measuring concentrations over a vertical plane orthogonal to the main flow at one or more fixed locations downstream (see Fig. 5.1).

A steady discharge is to be preferred to an instantaneous one because:

1. longitudinal concentration gradients are small, and longitudinal dispersion can be neglected;
2. a fairly long time interval is available to collect samples over each vertical plane.

Because of large eddies the tracer plume may move bodily around the vertical plane during the course of sampling. This has been corrected in past studies by collecting groups of samples simultaneously using a multiple sampling device and plotting concentrations on a *y–z* co-ordinate system whose origin is the centroid of each group of samples.

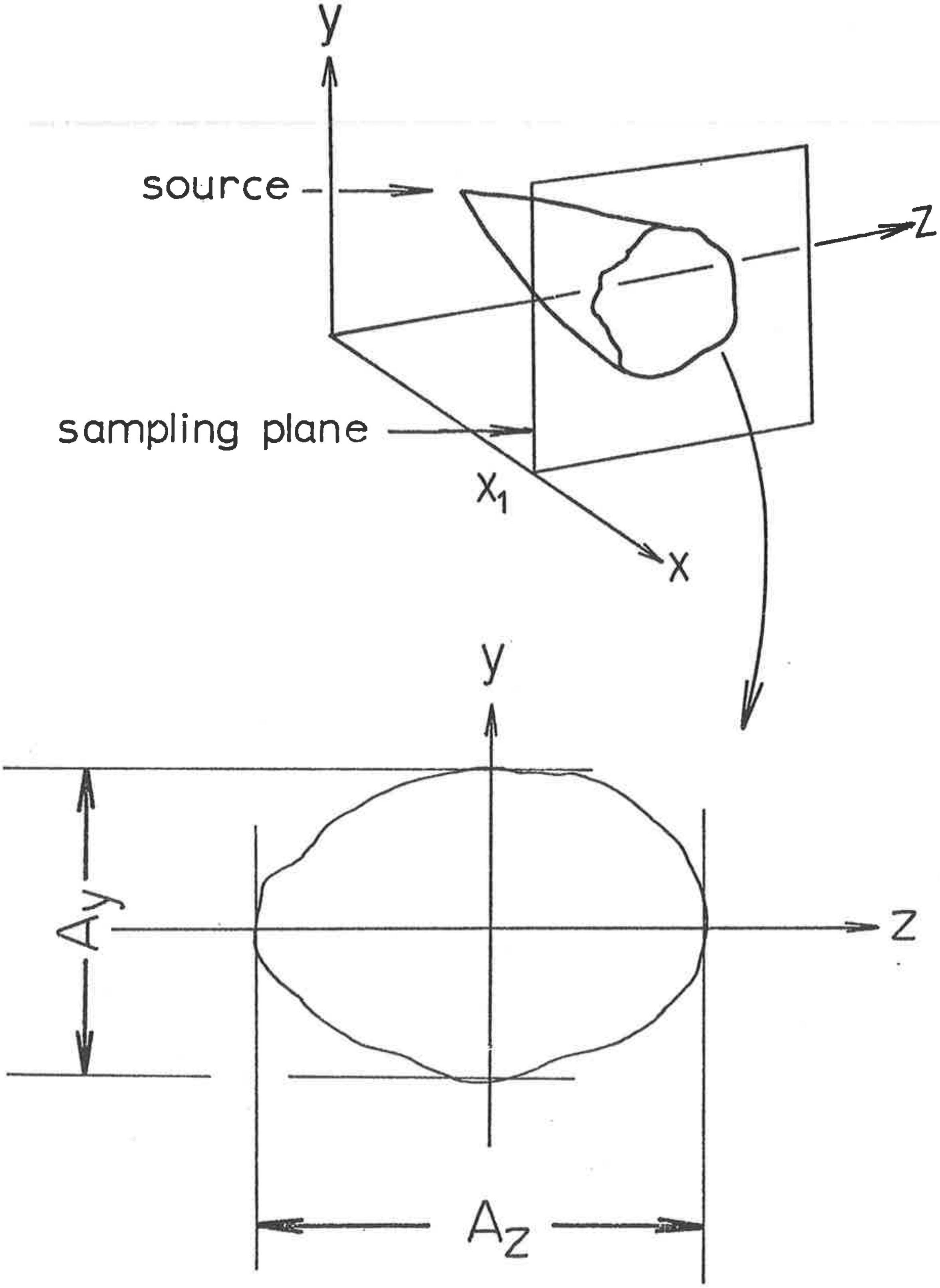


Figure 5.1 Measurements of vertical and transverse dispersion coefficients.

5.4.2 *Analytical Techniques* At each location lines of equal concentration are drawn, correcting where necessary for bodily movement of the centre of the plume. The rate at which tracer passes the sampling plane (the mass flux) is estimated and compared with the known injection rate to check for inaccuracies in measurement and interpolation. The flux is

$$q = U\bar{C}A \tag{5.2}$$

where q = mass flux in $g.s^{-1}$, $\bar{C}$ = average concentration determined over an area A orthogonal to the flow.

5.4.3 *Outfall distant from any boundary* If tracer does not impinge on any boundary (as sketched in Fig. 5.1) then the lengths of the major axis, A_z, and the minor axis, A_y, are measured for the concentration contour $C = C^*$. Then

$$D_z = \frac{A_z^2}{\frac{4x}{U} \ln\left(\frac{qA_z}{4\pi x D_z A_y C^*}\right)} \tag{5.3}$$

Equation 5.3 must be solved for D_z by successive substitutions. Then

$$D_y = D_z\left(\frac{A_y}{A_z}\right)^2 \tag{5.4}$$

If the sampling sites are located far enough below the outfall for tracer to become vertically mixed, but close enough so that tracer does not impinge on either bank, then transverse dispersion coefficients can be estimated from

$$D_z = \left(\frac{q}{UdC_p}\right)^2 \frac{U}{4\pi x} \tag{5.5}$$

where C_p = peak concentration measured at a distance x below the outfall. If in addition to C_p, concentration C is measured at transverse location z, then

$$D_z = (z - z_o)^2 \frac{U}{4x \ln(C_p/C)} \tag{5.6}$$

where z_o = transverse location of the outfall. It may be possible to use equation 5.6 for several values of C and z and deduce an average value of D_z. Also equation 5.6 does not require knowledge of the mass inflow rate, q.

5.4.4 *Outfall close to a boundary* If tracer impinges on any boundary then equations 5.3 to 5.6 must be modified to take account of reflection which may occur. In practice the most common situation is an outfall located on either the bed or bank, and only this problem is considered here.

For an outfall on the river bed (far distant from either bank), equation 5.3 becomes

$$D_z = \frac{A_z^2}{\frac{4x}{U} \ln\left(\frac{qA_z}{2\pi x D_z A_y C^*}\right)} \tag{5.7}$$

and equation 5.4 remains the same. As before, equation 5.7 must be solved iteratively and U must be known.

For an outfall on either bank, assuming complete vertical mixing, equation 5.5 becomes

$$D_z = \left(\frac{q}{2UdC_p}\right)^2 \frac{U}{4\pi x} \tag{5.8}$$

but equation 5.6 remains the same.

5.4.5 *Use of aerial photography* This is a convenient way to study transverse mixing in rivers. It is necessary to ensure that:

1. an object whose dimensions are known appears in each frame so that distances can be scaled accurately, e.g., a bridge or building;
2. samples are collected in various parts of the tracer plume at the same time as photographs are taken, so that colour intensities can be converted to concentrations. If such samples cannot be collected, a rough estimate of D_z can be made by assuming that the edge of the tracer plume visible in the photographs corresponds to some value of C/C_p (say 5%) and then using equation 5.6. As shown in example 5.4, the value of D_z derived in this manner is not greatly affected by the choice of the ratio C/C_p.

5.5 Longitudinal Mixing

5.5.1 *Field techniques* An accurate method of measuring longitudinal dispersion coefficients in a river is to inject instantaneously a known amount of tracer and to measure cross sectional average concentration versus time profiles at several downstream sites. The following points should be noted concerning experimental design.

1. In the "advective zone" concentration profiles are likely to be highly skewed and D_x will not be constant.
2. Unless dispersion in the "advective zone" is particularly important then all measuring sites should be located below the "advective zone". Equation 4.2 and Table 4.1 can be used to estimate its length.
3. At least two and ideally five measuring sites should be used, spaced at 20–50 times the width of the channel apart.
4. For an accurate estimate of the dispersion coefficient, sufficient tracer should be injected to give a peak concentration at the most downstream site of about 15–20 times the minimum detectable concentration. Thus

$$W > 15\text{-}20\, C_m A \sqrt{4\pi D_x x/U} \tag{5.9}$$

   where C_m = minimum detectable concentration, D_x = estimate of the dispersion coefficient (made from Table 4.2) and x = distance of the most downstream site below the outfall.
5. The mean velocity, U, is the time of passage of the centroid of the tracer profiles which can be estimated fairly accurately by the time of passage of the peak.
6. Cross sectional averaging should be weighted by flow so as to preserve mass discharges past the section. Sampling can therefore be concentrated in the main stream.

5.5.2 *Analytical techniques* The dispersion coefficient can be estimated very roughly from

$$D_x = \left(\frac{W}{AC_p}\right)^2 \frac{1}{4\pi t_p} \tag{5.10}$$

where C_p = concentration observed at time t_p.
As a check the profile observed at one site should be routed downstream to another site using the so called "frozen cloud" model (Fischer 1973).

$$C(x_2, t) = \int_{-\infty}^{\infty} \frac{C(x_1, \tau)}{\sqrt{4\pi D_x(t_2 - t_1)}} \exp \frac{-(x_2 - x_1 - U(t - \tau))^2}{4D_x(t_2 - t_1)} U d\tau \tag{5.11}$$

If necessary U and D_x can be adjusted to obtain a satisfactory fit between the predicted and observed concentration profiles at the downstream site (see worked example 5.6.5). Appendix 2 contains a mini-computer program for doing this.

5.5.3 *Use of velocity measurements* D_x can be estimated if the velocity distribution across the channel is known (e.g., from gauging data) using (Fischer 1973)

$$D_x \cong 0.30 \frac{\overline{u'^2} b^2}{4Ru^*} \tag{5.12}$$

where $\overline{u'^2}$ = average over the cross-section of the square of $u' = U - u$
where U = cross-section mean velocity and u = velocity at any point in the cross section.

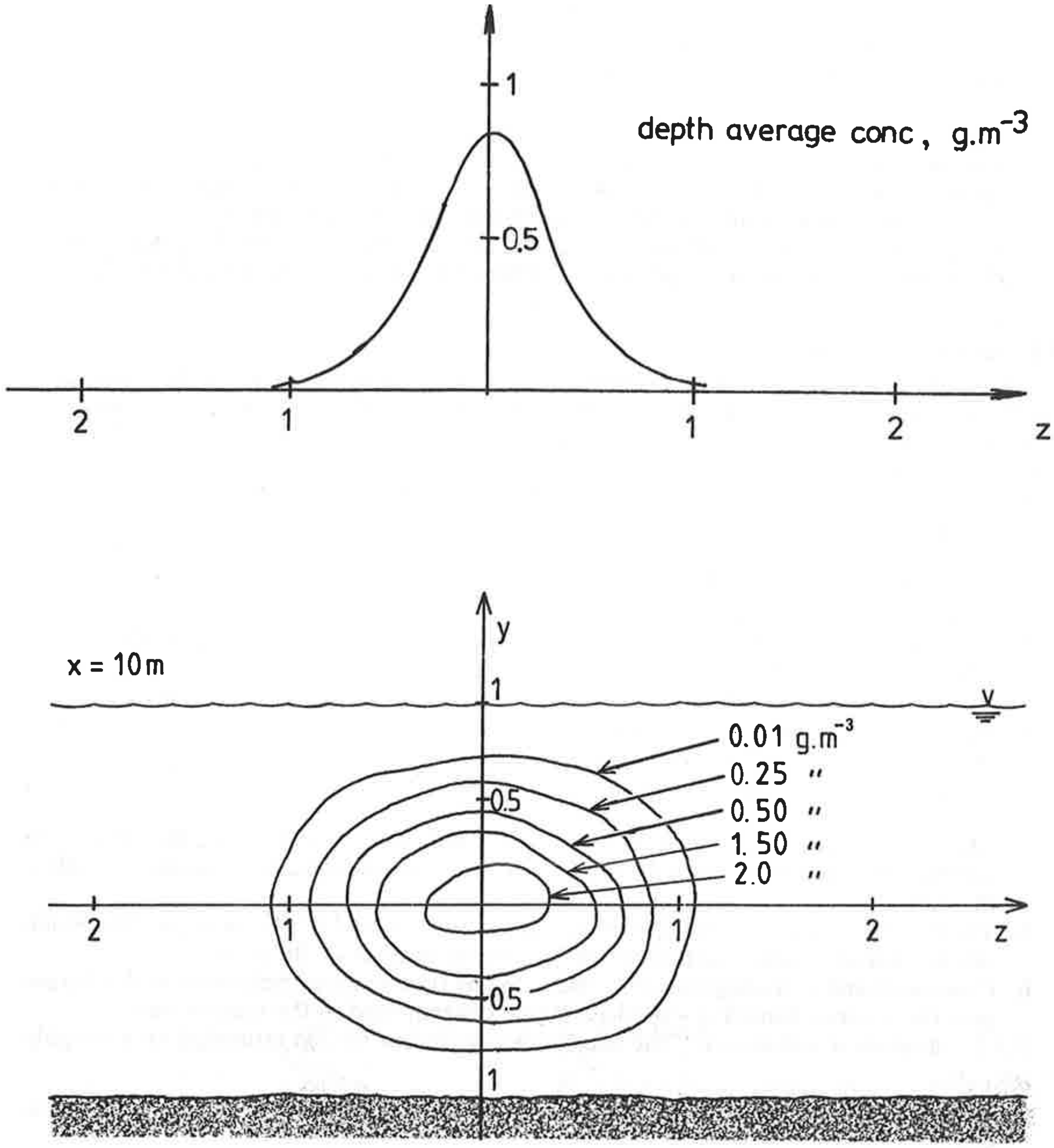


Figure 5.2 Observed concentration contours, examples 5.6.2 and 5.6.3.

5.6 Worked Examples

Example 5.6.1 Given that sites B and F in Fig. 4.1 were located 6.05 and 22.78 km below the injection point, estimate the mean velocity.

From Fig. 4.1 $t_1 = 2$ hours $t_2 = 9$ hours

From equation 5.1

$$U = (x_2 - x_1)/(t_2 - t_1)$$
$$= (22780\text{-}6050)/7 \times 3600$$
$$= 0.66 \text{ m.s}^{-1}$$

Example 5.6.2 Given the concentrations shown in Fig. 5.2 measured in a vertical plane a distance $x = 10$ m below a steady point source of tracer.

Take $U = 1.0 \text{ m.s}^{-1}$ derived from another experiment.

(a) Check that the mass flux approximates the 1 g.s^{-1} discharged.
(b) Estimate values of D_y and D_z.

(a) The average concentration measured $= 0.31 \text{ g.m}^{-3}$
The area covered $= 3.2 \text{ m}^2$
Thus flux $q = 0.31 \times 3.2 \times 1 \cong 1 \text{ g.s}^{-1}$

(b) Major axis $A_z = 1.1$ m
Minor axis $A_y = 0.75$ m
First guess $D_z = 100 \text{ cm}^2 \text{ s}^{-1}$
From equation 5.3 1st iteration $D_z = 64 \text{ cm}^2 \text{ s}^{-1}$
2nd $58 \text{ cm}^2 \text{ s}^{-1}$
3rd $57 \text{ cm}^2 \text{ s}^{-1}$
4th $57 \text{ cm}^2 \text{ s}^{-1}$

Thus $D_z = 57 \text{ cm}^2 \text{s}^{-1}$

From equation 5.4

$$D_y = D_z \left(\frac{A_y}{A_z}\right)^2 = 26 \text{ cm}^2.\text{s}^{-1}$$

Example 5.6.3 Use the depth average concentrations from Fig. 5.2 and estimate the value of D_z

(a) Given $q = 1 \text{ g.s}^{-1}$, $d = 2$ m and $U = 1 \text{ m.s}^{-1}$
(b) Given only that $U = 1 \text{ m.s}^{-1}$

(a) $C_p = 0.82 \text{ g.m}^{-3}$
Thus from equation 5.5

$$D_z = \left(\frac{q}{UdC_p}\right)^2 \frac{U}{4\pi x} = 30 \text{ cm}^2.\text{s}^{-1}$$

(b) $C = 0.01$ at $z - z_o = 1$ m
From equation 5.6

$$D_z = (z - z_o)^2 \frac{U}{4x\ln(C_p/C)} = 57 \text{ cm}^2.\text{s}^{-1}$$

$$C = 0.14 \text{ at } z - z_o = 0.6 \text{ m} \rightarrow D_z = 51 \text{ cm}^2.\text{s}^{-1}$$

and

Example 5.6.4 A tracer plume originating from a bank outfall is 25 m wide 1500 m downstream. $U = 0.5 \text{ m.s}^{-1}$. Estimate D_z.

From equation 5.6
assuming $C_p/C = 100$ then D $110 \text{ cm}^2 \text{ s}^{-1}$
50 $130 \text{ cm}^2 \text{ s}^{-1}$
20 $170 \text{ cm}^2 \text{ s}^{-1}$

Example 5.6.5 Use computer program ROUTE (see Appendix 2) to evaluate U and D_x from the dye test data collected in the Manawatu River listed below.

```
run
ROUTE    16:20           20-Jul-81

***************
PROGRAMME ROUTE
***************


DO YOU WANT A SYNOPSIS? Y


ROUTES A CONC V TIME PROFILE DOWN A UNIFORM CHANNEL
USING THE FROZEN CLOUD MODEL OF DISPERSION
THUS ALLOWING U & D VALUES TO BE ESTIMATED FROM FIELD DATA

USER MUST PRESCRIBE:  LOCATION OF INJECTION, U/S & D/S SITES
                      CONC V TIME PROFILES AT U/S & D/S SITES
THEN PROGRAMME ESTIMATES U & D VALUES
ROUTES THE PROFILE AT THE U/S SITE TO THE D/S SITE
GRAPHS THE OBSERVED & PREDICTED PROFILES AT THE D/S SITE

USER CAN ALTER U & D UNTIL A SATISFACTORY MATCH IS OBTAINED



ENTER LOCATION OF OUTFALL, KM               ? 75.00
SITE ID        ? site A
ENTER TIME OF DISCHARGE, HHMM               ? -0100

ENTER LOCATION OF U/S SITE, KM              ? 72.3
ENTER SITE ID ? site B
ENTER TIME V CONCENTRATION PROFILE
ENTER FILE NAME (NO EXT) FOR FILED DATA     ? MB

FILE HEADER    MANAWATU STATION B CONDENSED

DO YOU WANT AN INPUT DATA LISTING           ? Y
```

EXPLANATORY COMMENTS

Characters following a questionmark are entered by the user

Location of the point of tracer injection

Note the format : hours and minutes

Location of upstream sampling site

Data was previously stored in file MB.DAT

Short for YES

```
            TIME          CONC

1           -.333333       .18098
2            0             .492056
3            .166667      -.130096
4            .233333      -.130096
5            .333333      -.130096
6            .5            .077288
7            .666667      -.233788
8            .75          -.130096
9            .833333      -.130096
10           .9            7.60443
11           1             12.0019
12           1.06667       17.2991
13           1.08333       23.3285
14           1.11667       30.3177
15           1.16667       33.6604
16           1.2           37.003
17           1.23333       41.2573
18           1.28333       45.6635
19           1.30833       46.7271
20           1.35          47.3349
21           1.38333       47.9427
22           1.41667       47.031
23           1.5           45.2078
24           1.51667       44.2961
25           1.56667       44.2961
26           1.6           43.0806
27           1.68333       38.1173
28           1.76667       35.0785
29           1.83333       31.2294
30           1.93333       27.4309
31           2             23.9363
32           2.05          21.8091
33           2.25          15.7316
34           2.4           10.5848
35           2.5           8.65761
36           2.75          6.29881
37           2.85          4.27682
38           3             3.63738
39           3.25          2.77328
40           3.4           2.15113
41           3.5           1.73636
42           3.75          1.37344
43           4             .975755
44           4.25          .803132
45           5             .388364
46           5.5           .284672
47           6       .18098
48           6.5           .18098
49           7             .18098
50           8             .077288
51           9            -.130096
52           10            .077288
53           10.5          .077288
54           0             0
IS TIME IN HHMM FORMAT?
```

```
ENTER LOCATION OF D/S SITE, KM            ? 68.60 ←———— Location of downstream sampling site
ENTER SITE ID ? site D
ENTER TIME V CONCENTRATION PROFILE
ENTER FILE NAME (NO EXT) FOR FILED DATA   ? MD ←———— Data was stored previously in file MD.DAT

FILE HEADER   MANAWATU DYE TEST STATION D CONDENSED DATA

DO YOU WANT AN INPUT DATA LISTING         ? Y
                TIME           CONC

 1               2.5            0
 2               2.83333        2.04744
 3               2.93333        6.09142
 4               3.01667        10.6021
 5               3.08333        13.9484
 6               3.16667        20.7456
 7               3.21667        23.5311
 8               3.23334        23.9363
 9               3.28333        26.9751
 10              3.33333        25.4557
 11              3.36667        32.293
 12              3.4            32.7488
 13              3.46667        32.9513
 14              3.53333        33.7617
 15              3.58333        34.4707
 16              3.61667        34.1162
 17              3.7            33.9643
 18              3.75           33.3565
 19              3.81667        32.293
 20              3.91667        29.2542
 21              3.95           29.71
 22              3.96667        29.4061
 23              4.15           23.9363
 24              4.2            23.1766
 25              4.25           21.6066
 26              4.31667        21.2014
 27              4.38333        18.7704
 28              4.48333        15.8648
 29              4.95           9.51326
 30              5.11667        6.60988
 31              5.41667        4.43235
 32              5.5            3.39543
 33              5.65           3.29174
 34              6              2.25482
 35              6.25           1.66723
 36              6.5            1.26975
 37              6.75           .958672
 38              7              .906824
 39              7.5            .768568
 40              8              .492056
 41              8.5            .319236
 42              9              .284672
 43              9.5            .284672
 44              1              .232826
 45              10.5           .215544
 46              11             .232826
 47              11.5           .18098
 48              12             .077288
 49              12.5           .18098
 50              8              .077288
IS TIME IN HHMM FORMAT?
```

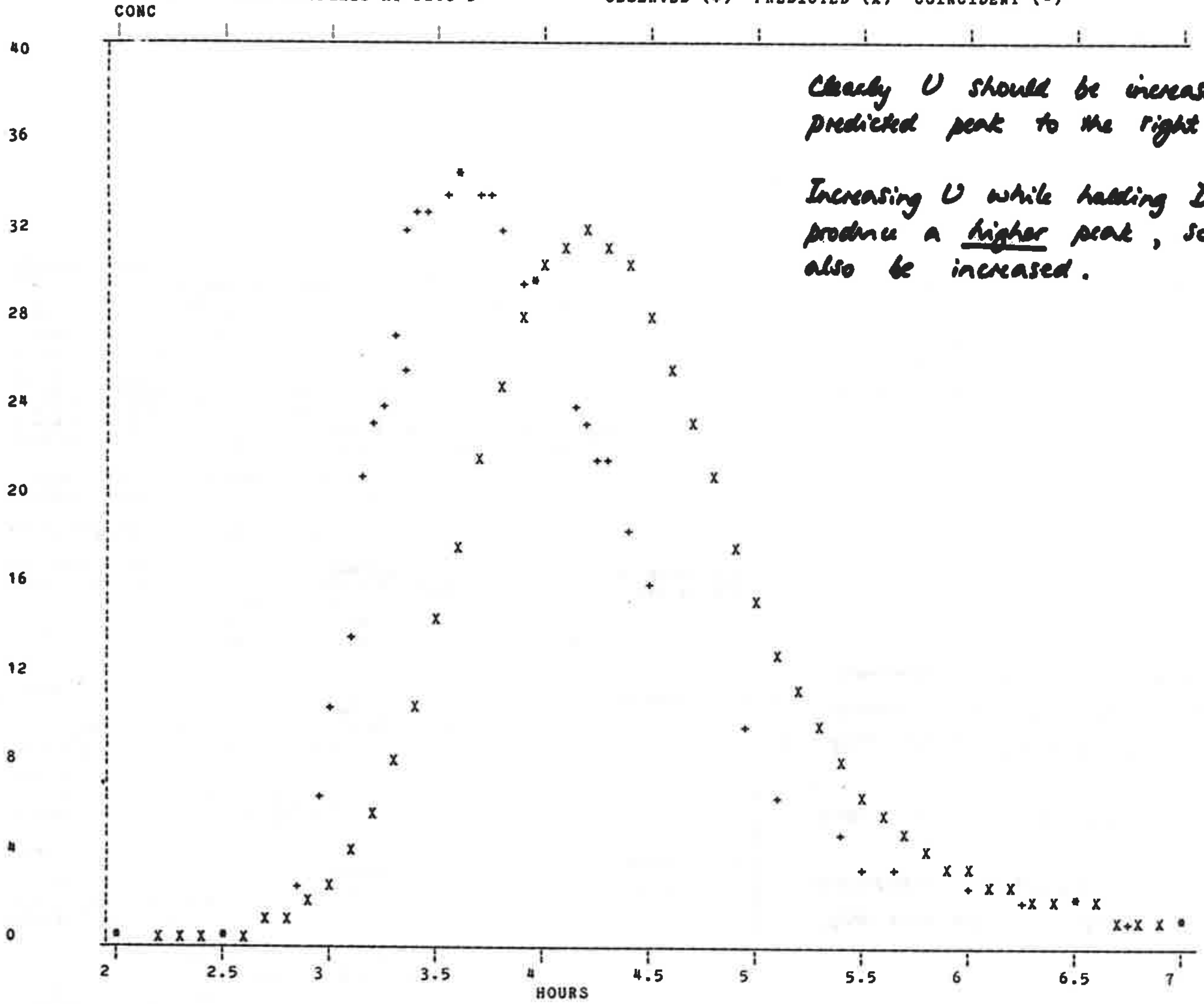
```
TIMES OF PEAK CONC
site A          -1  HR
site B           1.38333  HR   47.9427  MG/M^S
site D           3.58333       34.4707

VELOCITY ESTIMATES
site A TO     site B          1.13287  KM/HR
site A        site B          1.39636
site B        site D          1.68182
AVERAGE                       1.40368  KM/HR              .389912  M/S

INTEGRALS OF CONC V TIME PROFILES
site B           51.3216  MG.HR/M^3
site D           53.0549

DISPERSION COEFFICIENT ESTIMATES
site A TO     site B         .491045E-1  KM^2/HR
site A        site D         .080197
AVERAGE                      .646507E-1  KM^2/HR          17.9585  M^2/S


ENTER OUTPUT START TIME      ? 2.00
             FINISH TIME     ? 7.00
             NO. OF STEPS    ? 50
MEAN VELOCITY IS              1.40368  KM/HR              .389912  M/S
DISPERSION COEFFICIENT IS     .646507E-1  KM^2/HR         17.9585  M^2/S

ENTER OUTPUT FILE NAME       ?

OBSERVED  PEAK   34.4707    AT TIME      3.58333
ROUTED    PEAK   31.8992    AT TIME      4.2

DEFAULT DIMENSIONS FOR SKETCH OF CONC V TIME PROFILES
T MIN =  2.2  T MAX =  8    C MIN =  0    C MAX =  47.9427
ENTER C TO CHANGE ANY OF THESE             ? C
ENTER NEW VALUES <CR> RETAINS DEFAULT VALUE
T MIN          ? 2.00
T MAX          ? 7.00
C MIN          ? 0.00
C MAX          ? 40.00
T MIN =  2    T MAX =  7    C MIN =  0    C MAX =  40
ENTER C TO CHANGE ANY OF THESE             ?
ADVANCE PAPER?
```

These estimates are based on rates of downstream migration of peak concentration

Enable a check on mass conservation

Preliminary estimates based on a theoretical solution for a point discharge namely equation (4.2)

```
GRAPH  1      U= 1.40368  KM/HR   .389912  M/S          D= .646507E-1  KM^2/HR   17.9585  M^2/S
              TIME V CONC PROFILES AT site D             OBSERVED (+)  PREDICTED (X)  COINCIDENT (*)
```



Clearly U should be increased to shift the predicted peak to the right

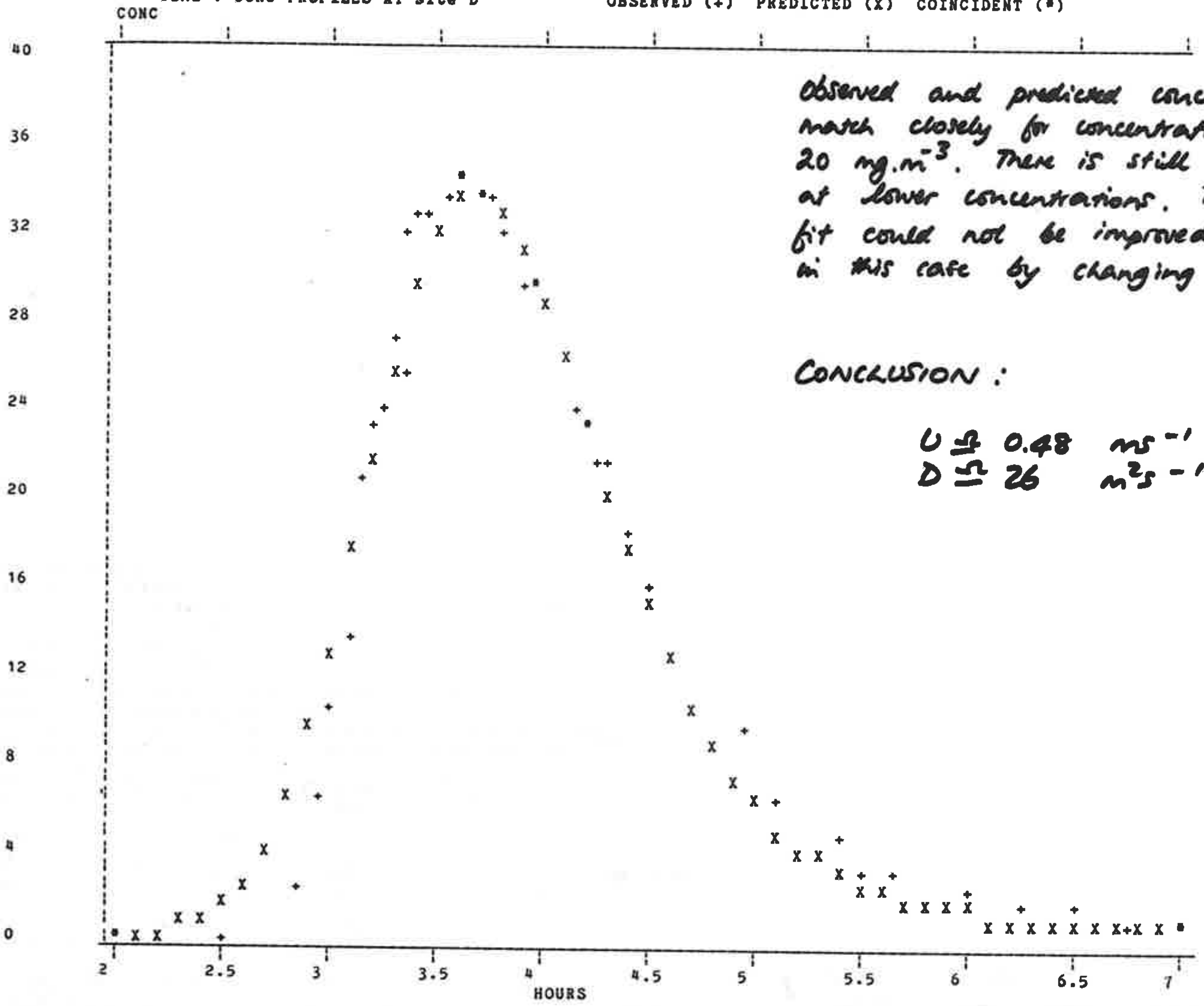
Increasing U while holding D constant will produce a higher peak, so D should also be increased.

```
DO YOU WANT TO CHANGE U & D ? Y
```

```
ENTER NEW VALUE TO CHANGE U OR D    <CR> RETAINS CURRENT VALUE
VELOCITY      M/S           ? 0.48
DISPERSION    M^2/S         ? 26

ENTER OUTPUT START TIME     ? 2.00
             FINISH TIME    ? 7.00
             NO. OF STEPS   ? 50
MEAN VELOCITY IS             1.728  KM/HR  .48  M/S
DISPERSION COEFFICIENT IS    .0936  KM^2/HR             26  M^2/S

ENTER OUTPUT FILE NAME      ?

OBSERVED  PEAK   34.4707    AT TIME      3.58333
ROUTED    PEAK   33.8784    AT TIME      3.7

DEFAULT DIMENSIONS FOR SKETCH OF CONC V TIME PROFILES
T MIN =  2    T MAX =  8    C MIN =  0    C MAX =  47.9427
ENTER C TO CHANGE ANY OF THESE             ? C
ENTER NEW VALUES <CR> RETAINS DEFAULT VALUE
T MIN         ?
T MAX         ? 7.00
C MIN         ?
C MAX         ? 40.00
T MIN =  2    T MAX =  7    C MIN =  0    C MAX =  40
ENTER C TO CHANGE ANY OF THESE             ?
ADVANCE PAPER?
```

} These values were selected after several trial runs

```
GRAPH  2      U= 1.728  KM/HR   .48  M/S  D= .0936  KM^2/HR   26  M^2/S
              TIME V CONC PROFILES AT site D            OBSERVED (+)  PREDICTED (X)  COINCIDENT (*)
```



Observed and predicted concentrations match closely for concentrations above 20 $mg.m^{-3}$. There is still a mismatch at lower concentrations. The overall fit could not be improved substantially in this case by changing U and/or D.

CONCLUSION :

$U \simeq 0.48 \ ms^{-1}$
$D \simeq 26 \ m^2s^{-1}$

```
DO YOU WANT TO CHANGE U & D ? N
```

Short for NO.

```
DO YOU WANT A DATA SUMMARY  ? N
ENTER OUTPUT FILE NAME      ?

Ready
```

observed and predicted profiles could be listed and filed by entering Y and a non-blank file name.

eg RUN1 entered in response to the last prompt would write an output file RUN1.SDT containing the observed and predicted profiles.

Entry of a blank file name exits from the programme.

6.0 ACKNOWLEDGEMENTS

The author is grateful to Dr T. F. W. Harris, University of Auckland and Dr A. G. Barnett, Hamilton Science Centre, Ministry of Works and Development, for their advice and encouragement during the preparation of this handbook; Professor I. R. Wood, University of Canterbury, made useful suggestions especially on the section describing the mechanisms of dispersion and on the problem of outfalls near boundaries.

7.0 REFERENCES

Chatwin, P. C. 1972: The cumulants of the distribution of a solution dispersing in solvent flowing through a tube. *Journal of Fluid Mechanics 51 (1)*: 63–7.

Elder, J. W. 1959: The dispersal of marked fluid in turbulent shear flow. *Journal of Fluid Mechanics 5*: 544–60.

El-Hadi, N. D. A.; Davar, K. S. 1976: Longitudinal dispersion for flow over rough beds. *Proceedings of the American Society of Civil Engineers 102* (*HY4*): 438–98.

Fischer, H. B. 1973: Longitudinal dispersion and turbulent mixing in open-channel flow. *Annual Review of Fluid Mechanics 5*: 59–78.

Fischer, H. B. 1976: Mixing and dispersion in estuaries. *Annual Review of Fluid Mechanics 8*: 107–33.

Harris, T. F. W. *pers. comm.*, University of Auckland.

Holley, E. R.; Abraham, G. 1973: Field tests on transverse mixing in rivers. *Proceedings of the American Society of Civil Engineers 99* (*HY12*): 2313–31.

Holly, F. M. 1975: Two-dimensional mass dispersion in rivers. *Hydraulics Paper No. 78,* Colorado State University.

Lau, Y. L.; Krishnappan, B. G. 1977: Transverse dispersion in rectangular channels. *Proceedings of the American Society of Civil Engineers 103* (*HY10*): 1173–89.

Liu, H. 1977: Predicting dispersion coefficients of streams. *Proceedings of the American Society of Civil Engineers 103* (*EE1*): 59–69

McQuivey, R. S.; Keefer, T. N. 1974: Simple method for predicting dispersion in streams. *Proceedings of the American Society of Civil Engineers 100* (*EE4*): 997–1011.

McQuivey, R. S.; Keefer, T. N. 1976: Dispersion—Mississippi River below Baton Rouge. *Proceedings of the American Society of Civil Engineers 102 (HY10)*: 1425–37.

Miller, A. C.; Richardson, E. V. 1974: Diffusion and dispersion in open channel flow. *Proceedings of the American Society of Civil Engineers 100* (*HY1*): 159–71.

Prych, E. A. 1970: Effects of density differences on lateral mixing in open channel flows. Report No. KH-R-21, California Institute of Technology.

Rutherford, J. C. 1979: Investigations of mechanisms affecting BOD concentrations in the Manawatu River near Palmerston North. Hamilton Science Centre, Internal Report, No. 79/24.

Rutherford, J. C.; Taylor M. E. U.; Davies J. D. 1980: Waikato River flushing rates. *Proceedings of the American Society of Civil Engineers 106* (*EE6*): 1131–50.

Sayre, W. W. 1968: Dispersion of mass in open-channel flow. *Hydraulics Paper No. 3* , Colorado State University.

Shen, H. T. 1973: Environmental impact on rivers — River mechanics III. Colorado State University.

Valentine, E. M. 1978: Effects of channel boundary roughness on longitudinal dispersion. PhD Thesis, University of Canterbury.

Ward, P. R. B. 1976: Measurements of estuary dispersion coefficients. *Proceedings of the American Society of Civil Engineers 102* (*EE4*): 855–60.

8.0 APPENDICES

8.1 Summary of equations

Assuming dispersion obeys Fick's Law, then the conservation of mass equation can be written (Fischer 1973)

$$\frac{\partial C}{\partial t} + u_i \frac{\partial C}{\partial x_i} = \frac{\partial}{\partial x_i}\left(D_i \frac{\partial C}{\partial x_i}\right) \tag{8.1}$$

where the co-ordinate directions for $i = 1, 2$ and 3 correspond to x, y and z, defined in Fig. 1.2. Equation 8.1 can be simplified in a river by assuming

$$u_2 = u_3 = 0 \tag{8.2}$$

$$\text{and} \quad u_1 = U \tag{8.3}$$

For a slug load of mass W released at $t = 0$, $x=y=z=0$ into an unbounded channel (Holly 1975; Shen 1973)

$$C(x,y,z,t) = W \frac{\exp\left(-\frac{(x-Ut)^2}{4D_x t}\right)}{\sqrt{4\pi D_x t}} \frac{\exp\left(-\frac{y^2}{4D_y t}\right)}{\sqrt{4\pi D_y t}} \frac{\exp\left(-\frac{z^2}{4D_z t}\right)}{\sqrt{4\pi D_z t}} \tag{8.4}$$

For a steady point discharge of mass q per unit time the superposition principle can be applied to equation 8.4. It is found that the effects of longitudinal dispersion are negligible. Thus (Holly 1975)

$$C(x,y,z) = q \frac{\exp\left(-\frac{y^2 U}{4D_y x}\right)}{\sqrt{4\pi D_y x}} \frac{\exp\left(-\frac{z^2 U}{4D_z x}\right)}{\sqrt{4\pi D_z x}} \tag{8.5}$$

This equation was used to draw Fig. 2.2.

For an instantaneous vertical line-source (Holly 1975)

$$C(x,z,t) = \frac{W}{d} \frac{\exp\left(-\frac{(x-Ut)^2}{4D_x t}\right)}{\sqrt{4\pi D_x t}} \frac{\exp\left(-\frac{z^2}{4D_z t}\right)}{\sqrt{4\pi D_z t}} \tag{8.6}$$

For an instantaneous transverse line source, simply exchange z for y and d for b in equation 8.6.

For a steady vertical line-source (Shen 1973)

$$C(x,z) = \frac{q}{d} \frac{\exp\left(\frac{Ux}{2D_x}\right)}{2\pi\sqrt{D_x D_z}} K_o\left(\frac{U}{2D_x}\sqrt{x^2 + \frac{D_x}{D_z} z^2}\right) \tag{8.7}$$

where K_o ( ) = modified Bessel function of the second kind. Clearly for a steady transverse line-source, simply exchange z for y and d for b in equation 8.7.

When longitudal dispersion is relatively unimportant (as frequently occurs in rivers), equation 8.7 simplifies to (Shen 1973)

$$C(x,z) = \frac{q}{d} \frac{\exp\left(-\frac{z^2 U}{4D_z x}\right)}{\sqrt{4\pi D_z x U}} \tag{8.8}$$

This equation was used to draw Fig. 2.1 and 3.1.

Boundaries affect the concentrations derived above. As a first approximation they behave like pure reflectors and the principle of images can be used. Thus when dealing with a point source located $y = \alpha$ and $z = \beta$ from the centreline of a channel, then

$$C(x,y,z,t) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} C(x, nd - \alpha + (-1)^n y, mb - \beta + (-1)^m z, t) \tag{8.9}$$

where each term on the right hand side must be evaluated using either equation 8.4 or 8.5.

For an instantaneous vertical line-source, complete vertical mixing is assumed and (Shen 1973)

$$C(x,z,t) = \sum_{m=-\infty}^{\infty} C(x, mb - \beta + (-1)^m z, t) \tag{8.10}$$

where the right hand side must be evaluated from equation 8.6.

For a steady vertical line-source

$$C(x,z) = \sum_{m=-\infty}^{\infty} C(x, mb - \beta + (-1)^m z) \tag{8.11}$$

where the right hand side is evaluated from equation 8.7. As before, equations 8.10 and 8.11 can be easily adapted to the case of instantaneous and steady transverse line-sources.

8.2 ROUTE computer program

Listing of a mini-computer program which can be used to estimate the velocity and the longitudinal dispersion coefficient from tracer concentration profiles measured at two sites in a river.

Program documentation

Program name: ROUTE

Programmer: J C. Rutherford
Hamilton Science Centre
Ministry of Works and Development
Private Bag
Hamilton

Date: January, 1980

Language: BASIS-PLUS

Computer: PDP 11/70 University of Waikato

Compatible computers: Those supporting extended BASIC

Format for input data: Either
- (i) free format entered interactively as directed by the program or
- (ii) from disc files created by previous runs of the program.

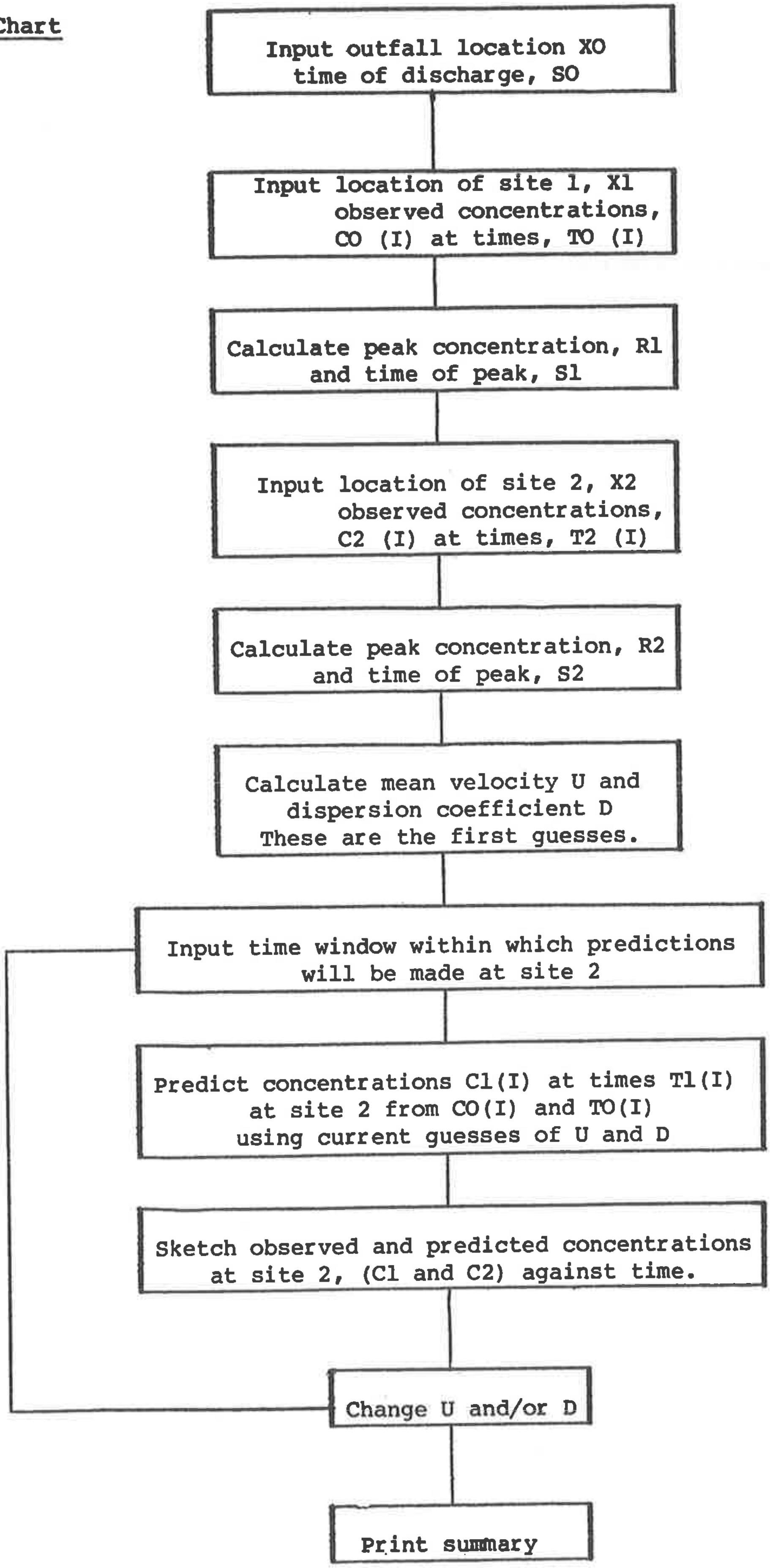
Users notes:
- (i) The program prints messages on the terminal which tell the operator the sequence of steps required to run the program.
- (ii) In most cases it is not possible to obtain a perfect fit between the observed and predicted profiles (see for example Rutherford *et al.*1980). The operator must decide when a satisfactory fit has been achieved.

Input/output example: See example 5.6.5, page 45-53

Program listing: Appended

Program flow chart: Appended

Flow Chart


Input outfall location XO
time of discharge, SO
Input location of site 1, X1
observed concentrations,
CO (I) at times, TO (I)
Calculate peak concentration, R1
and time of peak, S1
Input location of site 2, X2
observed concentrations,
C2 (I) at times, T2 (I)
Calculate peak concentration, R2
and time of peak, S2
Calculate mean velocity U and
dispersion coefficient D
These are the first guesses.
Input time window within which predictions
will be made at site 2
Predict concentrations C1(I) at times T1(I)
at site 2 from CO(I) and TO(I)
using current guesses of U and D
Sketch observed and predicted concentrations
at site 2, (C1 and C2) against time.
Change U and/or D
Print summary

```
ROUTE    15:51            20-Jul-81
10       EXTEND
11 !
20       ON ERROR GO TO 10000
25 !
30       DEF FNT(T) ! CONVERTS TIME IN HHMM TO HH.HH
35       T=T/60-2*INT(T/100)/3 \ FNT=T
40       FNEND
50 !
100      DEF FNX%(T7) ! PART OF GRAPHICS
102      X%=0.5+(T7-T5)*F1%/T8
104      IF X%<0% OR X%>F1% THEN X%=-1%
106      FNX%=X% \ FNEND
107 !
110          DEF FNY%(C7) ! PART OF GRAPHICS
112          Y%=0.5+(C7-C5)*F2%/C8
114          IF Y%<0% OR Y%>F2% THEN Y%=-1%
116          FNY%=Y% \ FNEND
117 !
118 !
119 !
120      DIM T0(220%),T1(220%),T2(220%),T3%(220%),T4%(220%),M1(20%),M2(20%),M3(20%)
130      DIM C0(220%),C1(220%),C2(220%),C3%(220%),C4%(220%),A%(220%),S$(100%)
140 !
150 !
1000     PRC=0.2 \ C$=',' \ N2%=0% \ L0=.1 \ F1%=100% \ F2%=50% \ Q%=0% \ F2$='.DAT'
1001 !
1002 !
1010     PRINT  \ PRINT'****************'\PRINT 'PROGRAMME ROUTE'\PRINT'****************'\PRINT
1020     PRINT  \ INPUT 'DO YOU WANT A SYNOPSIS';A$ \ IF A$='Y' THEN GOSUB 6000
1030 PRINT \ INPUT 'ENTER LOCATION OF OUTFALL, KM',X0 \ INPUT 'SITE ID',X0$
1035 INPUT 'ENTER TIME OF DISCHARGE, HHMM',S0 \ S0=FNT(S0)
1040 PRINT \ INPUT 'ENTER LOCATION OF U/S SITE, KM',X1 \ INPUT 'ENTER SITE ID',X1$
1045 PRINT 'ENTER TIME V CONCENTRATION PROFILE' \ GOSUB 11070
1050 N0%=N2%\R1=0\FOR I%=1 TO N0%\T0(I%)=T2(I%)\C0(I%)=C2(I%)\IF C0(I%)>R1 THEN R1=C0(I%)\S1=T0(I%)
1055 NEXT I%
1060 PRINT \ INPUT 'ENTER LOCATION OF D/S SITE, KM',X2 \ INPUT 'ENTER SITE ID',X2$
1065 PRINT 'ENTER TIME V CONCENTRATION PROFILE' \ GOSUB 11070
1070 R2=0 \ FOR I%=1% TO N2% \ IF C2(I%)>R2 THEN R2=C2(I%) \ S2=T2(I%)
1075 NEXT I%
1079 ! ENSURE DISTANCE INCREASES DOWNSTREAM
1080 IF X0>X2 THEN X0=500-X0 \ X1=500-X1 \ X2=500-X2
1081 PRINT
1084 ! PRINT SUMMARY
1085 PRINT 'TIMES OF PEAK CONC' \ PRINT X0$,S0' HR' \ PRINT X1$,S1' HR',R1' MG/M^S' \ PRINT X2$,S2,R2
1090 ! ESTIMATE VELOCITY
1095 U1=(X1-X0)/(S1-S0) \ U2=(X2-X0)/(S2-S0) \ U3=(X2-X1)/(S2-S1) \ U=(U1+U2+U3)/3 \ U0=U/3.6
1096 PRINT
1100 PRINT 'VELOCITY ESTIMATES' \ PRINT X0$' TO ',X1$,U1' KM/HR' \ PRINT X0$,X1$,U2 \ PRINT X1$,X2$,U3
1105 PRINT 'AVERAGE',,U' KM/HR',U0' M/S'
1200 ! ESTIMATE DISPERSION COEFFICIENT
1202 M1=0 \ FOR I%=2 TO N0% \ M1=M1+(C0(I%)+C0(I%-1))*(T0(I%)-T0(I%-1))/2 \ NEXT I%
1204 M2=0 \ FOR I%=2 TO N2% \ M2=M2+(C2(I%)+C2(I%-1))*(T2(I%)-T2(I%-1))/2 \ NEXT I%
1206 PRINT
1210 PRINT 'INTEGRALS OF CONC V TIME PROFILES' \ PRINT X1$,M1' MG.HR/M^3' \ PRINT X2$,M2
1215 D1=(U1*M1/R1)^2/4/PI/(S1-S0) \ D2=(U2*M2/R2)^2/4/PI/(S2-S0) \ D=(D1+D2)/2 \ D0=D/3.6E-03
1216 PRINT
1220 PRINT 'DISPERSION COEFFICIENT ESTIMATES' \ PRINT X0$' TO',X1$,D1' KM^2/HR' \ PRINT X0$,X2$,D2
1225 PRINT 'AVERAGE',,D' KM^2/HR ',D0' M^2/S'
1230 !
1235 ! ROUTE FROM U/S SITE TO D/S WITH SPECIFIED VALUES OF U & D
1240 !
1245 ! SPECIFY TIME WINDOWS
1250 !
1465 PRINT \ PRINT
1470    INPUT 'ENTER OUTPUT START TIME',P3$ \ IF P3$='' THEN 1510 ELSE P3=VAL(P3$)
1480    INPUT '               FINISH TIME',P5$ \ IF P5$='' THEN 1530 ELSE P5=VAL(P5$)
1490    INPUT '               NO. OF STEPS', P6% \ IF P6%=0% THEN P6%=200%
1500    P4=(P5-P3)/P6% \ GO TO 1550
1510    INPUT '               TIME STEP',P4 \ IF P4=0 THEN P4=T0(2%)-T0(1%)
1520    P3=T0(1%)+P4*INT((P9+SQR(3*P2))/P4) \ GO TO 1540
1530    INPUT '               TIME STEP',P4 \ IF P4=0 THEN P4=T0(2%)-T0(1%)
1540    P6%=220%
1550 PRINT 'MEAN VELOCITY IS ',U' KM/HR',U0' M/S'
1560    PRINT 'DISPERSION COEFFICIENT IS ',D' KM^2/HR ',D0' M^2/S'
1561 !
1562 ! SEGMENT WHICH DOES THE ROUTING
1563 !
1565    P2=4*D*(X2-X1)/U \ X9=X2-X1
1570    FOR I%=1% TO N0% \ IF C0(I%)<L0 THEN C0(I%)=0
1580    NEXT I%
1590    L1%=0% \ Q%=Q%+1% \ R3=0
1600    FOR L%=1% TO P6%+1%
1610    R0=0 \ S5=P3+(L%-1%)*P4
1620    FOR M%=2% TO N0%
1630    E0=(X9-U*(S5-(T0(M%)+T0(M%-1%))/2))^2/P2
1640    IF E0<35 THEN R0=R0+(C0(M%)+C0(M%-1%))*(T0(M%)-T0(M%-1%))*EXP(-E0)/2
1650    NEXT M%
1660    R0=R0*U/SQR(PI*P2)
1670    IF R0<L0 THEN IF L1%>0% THEN 1730 ELSE 1690
1680    L1%=L1%+1% \ T1(L1%)=S5 \ C1(L1%)=R0
1690    IF R0>R3 THEN S3=S5 \ R3=R0
1720    NEXT L%
1730    N1%=L1%
1735 !
1740 ! OPTION TO STORE THE ROUTED PROFILE
1745 !
1870    PRINT  \ INPUT 'ENTER OUTPUT FILE NAME',A$ \ IF A$="" THEN 1960
1880    A$=A$+F2$ \ OPEN A$ FOR OUTPUT AS FILE 4%
1890    INPUT 'ENTER OUTPUT FILE HEADER',H$
1900    PRINT #4%, H$;",TIME,CONC"
```

```
1910     FOR I%=1% TO N1%
1920     PRINT #4%, T1(I%),C$,C1(I%) \ NEXT I%
1930     CLOSE 4%
1940 !
1945 ! PRINT SUMMARY OF THE LATEST ROUTING
1950 !
1960      PRINT  \ PRINT 'OBSERVED  PEAK  'R2,'AT TIME    'S2
1970      PRINT 'ROUTED    PEAK   'R3,'AT TIME    'S3 \ PRINT
2000 !
2010 ! GRAPHIC ROUTINE TO SKETCH ROUTED & OBSERVED PROFILES
2020 !
2170      T5=T1(1%) \ IF T2(1%)<T5 THEN T5=T2(1%)
2180      T6=T1(N1%) \ IF T2(N2%)>T6 THEN T6=T2(N2%)
2190      C5=0 \ C6=Q1 \ IF R1>C6 THEN C6=R1
2195      PRINT 'DEFAULT DIMENSIONS FOR SKETCH OF CONC V TIME PROFILES'
2200      PRINT 'T MIN = ';T5,'T MAX = ';T6,'C MIN = ';C5,'C MAX = ';C6
2210      INPUT 'ENTER C TO CHANGE ANY OF THESE',A$
2220      IF ASCII(A$)<>67% THEN 2280
2225      PRINT 'ENTER NEW VALUES <CR> RETAINS DEFAULT VALUE'
2230      INPUT 'T MIN',T7$ \ IF T7$>'' THEN T5=VAL(T7$)
2240      INPUT 'T MAX',T7$ \ IF T7$>'' THEN T6=VAL(T7$)
2250      INPUT 'C MIN',C7$ \ IF C7$>'' THEN C5=VAL(C7$)
2260      INPUT 'C MAX',C7$ \ IF C7$>'' THEN C6=VAL(C7$)
2270      GO TO 2200
2280      T8=T6-T5 \ C8=C6-C5
2290      !
          FORM PRINT POSNS

2300      FOR I%=1% TO N1%
2310      T3%(I%)=FNX%(T1(I%)) \ C3%(I%)=FNY%(C1(I%))
2360      NEXT I%
2370      FOR I%=1% TO N2%
2380      T4%(I%)=FNX%(T2(I%)) \ C4%(I%)=FNY%(C2(I%))
2430      NEXT I%
2440      !
          SORT

2450      A%(I%)=T3%(I%)+200%*C3%(I%) FOR I%=1% TO N1%
2460      FOR I%=1% TO N1%-1% \ A1%=0%
2470      FOR J%=I% TO 1% STEP -1% \ IF A1% THEN 2500 ELSE A1%=-1%
2480      IF A%(J%)<A%(J%+1%) THEN P=A%(J%) \ A%(J%)=A%(J%+1%) \ A%(J%+1%)=P \ A1%=0%
2490      NEXT J%
2500      NEXT I%
2510      FOR I%=1% TO N1% \ C3%(I%)=A%(I%)/200% \ T3%(I%)=A%(I%)-C3%(I%)*200% \ NEXT I%
2520      A%(I%)=T4%(I%)+200%*C4%(I%) FOR I%=1% TO N2%
2530      FOR I%=1% TO N2%-1% \ A1%=0%
2540      FOR J%=I% TO 1% STEP -1% \ IF A1% THEN 2570 ELSE A1%=-1%
2550      IF A%(J%)<A%(J%+1%) THEN P=A%(J%) \ A%(J%)=A%(J%+1%) \ A%(J%+1%)=P \ A1%=0%
2560      NEXT J%
2570      NEXT I%
2580      FOR I%=1% TO N2% \ C4%(I%)=A%(I%)/200% \ T4%(I%)=A%(I%)-C4%(I%)*200% \ NEXT I%
2590      !
          PLOT POINTS

2600      INPUT 'ADVANCE PAPER';A$
2610      PRINT 'GRAPH ';Q%,'U='U' KM/HR  'UO' M/S','D='D' KM^2/HR  'DO' M^2/S'
2630 PRINT,'TIME V CONC PROFILES AT 'X2$,'OBSERVED (+)  PREDICTED (X)  COINCIDENT (*)'
2640      S%=9% \ J%=1% \ K%=1% \ PRINT TAB(S%);' CONC'
2650      PRINT TAB(G%);'¦'; FOR G%=10% TO F1%+10% STEP 10% \ PRINT CHR$(13%);TAB(S%);
2660      PRINT '_'; FOR G%=0% TO F1%+1%
2670      FOR I%=F2% TO 0% STEP -1%
2680           S$(G%)=' ' FOR G%=0% TO F1%
2685           FOR I1%=J% TO 220%
2690                IF C3%(I1%)<>I% THEN 2725 ELSE IF T3%(I1%)<0% THEN 2720
2695 IF T3%(I1%)>100% THEN 2720
2700                IF S$(T3%(I1%))=' ' THEN S$(T3%(I1%))='X' ELSE S$(T3%(I1%))='*'
2720           NEXT I1%
2725           J%=I1% \ FOR I1%=K% TO 220%
2730                IF C4%(I1%)<>I% THEN 2770 ELSE IF T4%(I1%)<0 THEN 2760
2735 IF T4%(I1%)>100% THEN 2760
2740                IF S$(T4%(I1%))=' ' THEN S$(T4%(I1%))='+' ELSE S$(T4%(I1%))='*'
2760           NEXT I1%
2770           K%=I1% \ PRINT \ IF 1*I%/5>I%/5% THEN 2800 ELSE C=C5+I%*C8/F2%
2780           IF ABS(C)<ABS(C8)/10000 THEN C=0
2790           PRINT LEFT(NUM1$(C),5%);
2800           PRINT TAB(S%);'¦';
2810           PRINT TAB(L%+S%+1%);S$(L%);  UNLESS S$(L%)=' ' FOR L%=0% TO F1%
2820      NEXT I%
2830      PRINT CHR$(13%);TAB(S%); \ PRINT '_'; FOR G%=0% TO F1%+2% \ PRINT
2840      PRINT TAB(S%+J%+1%);'¦'; FOR J%=0% TO F1% STEP 10% \ PRINT
2850      FOR J%=0% TO F1% STEP 10%
2860           T=T5+J%*T8/F1% \ IF ABS(T)<ABS(T8)/10000 THEN T=0
2870           PRINT TAB(S%+J%);LEFT(NUM1$(T),5%);  \ NEXT J% \ PRINT
2880      PRINT TAB(50%);'HOURS' \ PRINT  \ PRINT
2890      M1(Q%)=U \ M2(Q%)=D
2900 !
2910 ! ALTER U &/OR D IF REQUIRED
2920 !
2930                INPUT 'DO YOU WANT TO CHANGE U & D',A$
2940                PRINT \ IF ASCII(A$)<>89% THEN 5000
2945 GOSUB 12945
2950 GO TO 1470
3000 !
3010 ! PRINT A GRAND SUMMARY IF REQUIRED
3020 !
5000      PRINT \ INPUT 'DO YOU WANT A DATA SUMMARY',D$
5010      IF ASCII(D$)<>89% THEN 5220
5020      INPUT 'ADVANCE PAPER',D$
5030      PRINT 'PROGRAMME ROUTE - SUMMARY' \ PRINT ,X1$,,X2$;
5040      IF N2%=0% THEN PRINT  \ GO TO 5060
5050      PRINT ,,X2$
```

```
5060      PRINT ,'X = ';X1,,'X = ';X2 \ PRINT ,'INITIAL',,'PREDICTED';
5070      IF N2%=0% THEN PRINT  \ GO TO 5090
5080      PRINT ,,'OBSERVED'
5090      PRINT ,'TIME','CONC','TIME','CONC';
5100      IF N2%=0% THEN PRINT  \ GO TO 5120
5110      PRINT ,'TIME','CONC'
5120      N4%=N0% \ IF N1%>N4% THEN N4%=N1% \ IF N2%>N4% THEN N4%=N2%
5130      FOR I%=1% TO N4%
5140      PRINT I%,
5150      IF I%>N0% THEN PRINT ,, \ GO TO 5170
5160      PRINT T0(I%),C0(I%),
5170      IF I%>N1% THEN PRINT ,, \ GO TO 5190
5180      PRINT T1(I%),C1(I%),
5190      IF I%>N2% THEN PRINT  \ GO TO 5210
5200      PRINT T2(I%),C2(I%)
5210      NEXT I% \ PRINT
5220      PRINT \ INPUT 'ENTER OUTPUT FILE NAME',A$ \ IF A$="" THEN 5310
5230      A$=A$+F2$ \ OPEN A$ FOR OUTPUT AS FILE 4%
5240      INPUT 'ENTER OUTPUT FILE HEADER',H$
5250      PRINT #4%, H$;",TIME,CONC"
5260      FOR I%=1% TO N1%
5270      PRINT #4%, T1(I%),C$,C1(I%) \ NEXT I%
5280 CLOSE 4%
5310 GO TO 32760
5320      !
6000 ! SYNOPSIS
6001 PRINT \ PRINT \ PRINT \ PRINT \ PRINT
6005 PRINT 'ROUTES A CONC V TIME PROFILE DOWN A UNIFORM CHANNEL'
6006 PRINT 'USING THE FROZEN CLOUD MODEL OF DISPERSION'
6010 PRINT 'THUS ALLOWING U & D VALUES TO BE ESTIMATED FROM FIELD DATA'
6030 PRINT
6040 PRINT 'USER MUST PRESCRIBE:  LOCATION OF INJECTION, U/S & D/S SITES'
6050 PRINT '                      CONC V TIME PROFILES AT U/S & D/S SITES'
6060 PRINT 'THEN PROGRAMME ESTIMATES U & D VALUES'
6070 PRINT 'ROUTES THE PROFILE AT THE U/S SITE TO THE D/S SITE'
6080 PRINT 'GRAPHS THE OBSERVED & PREDICTED PROFILES AT THE D/S SITE'
6085 PRINT
6090 PRINT 'USER CAN ALTER U & D UNTIL A SATISFACTORY MATCH IS OBTAINED'
6091 PRINT \ PRINT \ PRINT \ PRINT \ PRINT
7000 RETURN
10000    IF ERR<>11% THEN 11000
10010    IF ERL=11150 THEN RESUME 11170
11000    ON ERROR GO TO

11070    INPUT 'ENTER FILE NAME (NO EXT) FOR FILED DATA',A$ \ IF A$='' THEN 11190 ELSE A$=A$+F2$
11120    OPEN A$ FOR INPUT AS FILE 1%
11130    INPUT #1%, B$ \ B1%=INSTR(1%,B$,C$) \ IF B1% THEN B$=LEFT(B$,B1%-1%)
11140 PRINT \ PRINT 'FILE HEADER',B$ \ PRINT \ I%=0%
11150    I%=I%+1% \ INPUT #1%, T2(I%),C2(I%) \ GO TO 11150
11170    N2%=I% \ CLOSE 1%
11180    GO TO 11320

11190    PRINT  \ PRINT 'ENTER TIME AND CONC INPUT DATA SEPARATED BY COMMAS'
11200    FOR I%=1% TO 220% \ PRINT I%, \ INPUT LINE D$ \ IF LEN(D$)=2 THEN 11250
11205    D$=CVT$$(D$,4%)
11210    Z0%=INSTR(1%,D$,C$)
11220    T2(I%)=VAL(LEFT(D$,Z0%-1%))
11230    C2(I%)=VAL(MID(D$,Z0%+1%,100%))
11240    NEXT I%
11250    N2%=I%-1%
11260    INPUT 'IF YOU WANT TO FILE THIS DATA, ENTER FILE NAME',A$
11270    IF A$="" THEN 11320 ELSE A$=A$+F2$
11280    OPEN A$ FOR OUTPUT AS FILE 2%
11290    INPUT 'ENTER FILE HEADER',H$ \ PRINT #2%, H$;",TIME,CONC"
11300    FOR I%=1% TO N2% \ PRINT #2%, T2(I%),C$,C2(I%)
11310    NEXT I% \ CLOSE 2%
11320    INPUT 'DO YOU WANT AN INPUT DATA LISTING',B$
11330    IF ASCII(B$)<>89% THEN 11360
11340    PRINT ,'TIME','CONC' \ PRINT
11350    FOR I%=1% TO N2% \ PRINT I%,T2(I%),C2(I%) \ NEXT I%
11360 !
11361 INPUT 'IS TIME IN HHMM FORMAT';A$ \ IF LEN(A$)=0 THEN 11370
11362 FOR I%=1% TO N2% \ T2(I%)=FNT( T2(I%) )
11363 NEXT I% \ GO TO 11320
11370 !
12000 RETURN
12945              PRINT 'ENTER NEW VALUE TO CHANGE U OR D    <CR> RETAINS CURRENT VALUE'
12950 INPUT 'VELOCITY','M/S',U$ \ IF U$='' THEN 12955
12951 U0=VAL(U$) \ U=U0*3.6 \ GO TO 12960
12955 INPUT 'VELOCITY','KM/HR',U$ \ IF U$='' THEN 12960
12956 U=VAL(U$) \ U0=U/3.6
12960 INPUT 'DISPERSION','M^2/S',D$ \ IF D$='' THEN 12965
12961 D0=VAL(D$) \ D=D0*3.6E-03
12962 GO TO 12980
12965 INPUT 'DISPERSION','KM^2/HR',D$ \ IF D$='' THEN 12980
12966 D=VAL(D$) \ D0=D/3.6E-03
12980             PRINT \ RETURN
32760   CLOSE 1%,2%,3%,4% \ PRINT
32767   END

Ready
```

P. D. HASSELBERG, GOVERNMENT PRINTER, WELLINGTON, NEW ZEALAND—1983

93074J—PT

WATER AND SOIL MISCELLANEOUS PUBLICATIONS

1. Rainfalls and floods of Cyclone Alison, March 1975, on the north-eastern Ruahine Range. P. J. Grant, N. V. Hawkins, W. Christie. ($1) 1978
2. Water quality research in New Zealand 1977. Sally F. Davis ($2.50) 1978
3. Liquid and waterborne wastes research in New Zealand 1977. S. F. Davis ($2) 1978
4. Synthetic detergents working party report. ($1) 1978
5. Water quality control committee report. ($1) 1978
6. Suggestions for developing flow recommendations for in-stream uses of New Zealand streams. J. C. Fraser. ($1) 1978
7. Index to hydrological recording stations in New Zealand 1978. ($2) 1978
8. Water rights for the Clyde Dam, Clutha hydro power development. ($1.50) 1979
9. Index to hydrological recording stations in New Zealand 1979. ($2) 1979
10. Water quality research in New Zealand 1978. Denise F. Church. ($3) 1980
11. Liquid and waterborne wastes research in New Zealand 1978. D. F. Church. ($2) 1980
12. Catchment register for New Zealand, Volume 1. ($8) 1981
13. New Zealand recreational river survey. Pt 1: Introduction. G. D. and J. H. Egarr. ($5) 1981
14. New Zealand recreational river survey. Pt 2: North Island rivers. G. D. and J. H. Egarr. ($5) 1981
15. New Zealand recreational river survey. Pt 3: South Island rivers. G. D. and J. H. Egarr. ($12) 1981
16. Waimea East irrigation scheme information booklet. (Out of stock) 1980
17. Hawke's Bay area planning study: Urban capability assessment. ($4) 1980
18. Index to hydrological recording stations in New Zealand 1980. ($2) 1980
19. Rakaia water use and irrigation development. D. R. Maidment, W. J. Lewthwaite, S. G. Hamblett. ($3) 1980
20. Water quality research in New Zealand 1979. B. J. Biggs. ($4) 1980
21. Liquid and waterborne wastes research in New Zealand 1979. B. J. Biggs. ($2) 1980
22. Baseline water quality of the Manawatu water region 1977–78. K. J. Currie, B. W. Gilliland. ($3) 1980
23. Effects of land use on water quality—a review. R. H. S. McColl and Helen R. Hughes. ($5) 1981
24. Summaries of water quality and mass transport for Lake Taupo catchment, New Zealand. C. J. Schouten, W. Terzaghi, Y. Gordon. ($5) 1981
25. The report of the water quality criteria working party. ($3) 1981
26. Handbook on mixing in rivers. J. C. Rutherford ($8) 1981
27. Index to hydrological recording stations in New Zealand 1981. ($2) 1981
28. Bibliography of oceanography and sedimentology for the Northland-Auckland coast. T. F. W. Harris and T. Hume. ($3) 1981
29. Aquatic oxygen seminar proceedings. Hamilton, November 1980. ($10) 1982
30. Future groundwater research and survey in New Zealand. ($3) 1982
31. Land and water resource surveys of New Zealand: map coverage and reference lists. C. L. Clark. ($10) 1982
32. A procedure for characterising river channels. M. P. Mosley. ($8) 1982
33. The United States Environmental Protection Agency's 1980 ambient water quality criteria: a compilation for use in New Zealand. D. G. Smith. ($5) 1982
34. Water quality research in New Zealand, 1981. J. S. Gifford. ($5) 1982
35. Liquid and waterborne wastes research in New Zealand, 1981. J. S. Gifford. ($3) 1982
36. New Zealand river temperature regimes. M. P. Mosley. ($8) 1982
37. Landslip and flooding hazards in Eastbourne Borough—a guide for planning. ($8) 1982
38. Physical and chemical methods for water quality analysis. ($5) 1982
39. A guide to the common freshwater algae in New Zealand. ($5) 1982
40. Peatlands policy study; reports and recommendations. ($5) 1982
41. Index to hydrological recording stations in New Zealand 1982. ($5) 1982
42. A draft for a national inventory of wild and scenic rivers: Part 1, nationally important rivers. ($2) 1982
43. A review of land potential in the Bay of Plenty—Volcanic Plateau region. ($10) 1982
44. An approach to stormwater management planning. ($5) 1982
45. Catchment management for optimum use of land and water resources: Documents from an ESCAP seminar. Part 1—Introductory and country statements. ($10) 1982
46. Catchment management for optimum use of land and water resources: Documents from an ESCAP seminar. Part 2—New Zealand contributions. ($10) 1982
47. River Low Flows: Conflicts of water use. ($5) 1982
48. Catchment control in New Zealand. A. L. Poole. ($15) 1983
49. River and Estuary mixing Workshop; Hamilton. ($5) 1983