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Sensitivity of management reference points to the ratio of B_{msy}/B_0 determined by the Pella-Tomlinson shape parameter fitted to New Zealand rock lobster data

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Executive Summary

The objective of this paper is to show that the arbitary choice of the Fox model for the assessment of New Zealand rock lobster fishery may cause significant errors in the estimated management quantities. As a direct consequence of the results in this paper the model used for assessing New Zealand rock lobster has changed to reflect the biology of these fisheries.

1 Introduction

Surplus production modelling has been used in the assessment of the New Zealand red rock lobster (*Jasus edwardsii*) fishery (Breen & Kendrick 1994) and in many other fisheries where there are insufficient data for a fully age-structured analysis. New Zealand stock assessments generally calculate reference points such as maximum sustainable yield (*msy*) and biomass at maximum sustainable yield (B_{msy}) to provide advice on which to base management decisions (Annala 1993a). However, surplus production models have B_{msy} fixed as part of their formulation because of the underlying mathematical assumptions, a characterisitc which is often overlooked when choosing such a model. This may lead to a particular model not matching the actual stock dynamics. The following table summarises each of the most commonly used surplus production models and their corresponding assumption regarding B_{msy}. A yield curve calculated using each of these models is presented in Figure 1.

Model	$B_{msy} = \%B_0$	Reference
Schaefer	Fixed at 50%	Schaefer (1954)
Fox	Fixed at 37%	Fox (1975)
Pella-Tomlinson	Estimated or fixed at any level	Pella and Tomlinson (1969)

The Fox model has historically been chosen to assess the N.Z. rock lobster stock (Breen & Kendrick 1994).

The Pella-Tomlinson (P-T) model has a shape parameter that sets the position of B_{msy} relative to B_0 at any arbitrary point. However, fishery data are not usually informative enough to estimate the P-T shape parameter and it is usually better to set this parameter to a value which is consistent with the known stock dynamics.

The surplus production model used in any assessment will therefore determine B_{msy}, and this will in turn affect the assessment of the position of the stock relative to that which will produce *msy*. The purpose of this paper is to test the sensitivity of the estimated parameters and the reference points which are normally calculated during a stock assessment to the P-T shape parameter (using the N.Z. rock lobster data). Implicit in this analysis is a test of the sensitivity of the conclusions of the N.Z. rock lobster stock assessment to the underlying assumptions being made regarding the position of B_{msy} relative to B_0.

2 Methods

2.1 Equations

Gilbert's (1992) formulation of the Pella-Tomlinson model is used to investigate the sensitivity of reference points to the assumption made about the position of B_{msy} in respect to B_0.

The Schaefer (1954) model can be written in the form:

$$g(B_y) = rB_y - \frac{r}{k}B_y^2 \qquad \textbf{(Eq. 1)}$$

$$B_{y+1} = B_y + g(B_y) - C_y \qquad \textbf{(Eq. 2)}$$

Where:

- B_y is the biomass in year y
- r is the intrinsic growth rate parameter
- k is the average unexploited equilibrium biomass ($=B_0$)
- C_y is the catch in year y

Pella & Tomlinson (1969) suggested that the Schaefer model could be modified by raising B_y to the power of m to allow skewness in the production function (Hilborn & Walters 1992).

$$g(B_y) = rB_y - \frac{r}{k}B_y^m \qquad \textbf{(Eq. 3)}$$

where m is the Pella-Tomlinson shape parameter.

The problem with this formulation is that when $m \neq 2$, k is no longer the unfished equilibrium population size.

Gilbert (1992) used a discrete version of Pella and Tomlinson's original model (Eq. 4a) to derive consistent formulations of B_0 and r in terms of all values of m.

$$g(B_y) = HB_y^{\,m} - KB_y \qquad \textbf{(Eq. 4a)}$$

where H and K are parameters.

This was done by setting the yield [$g(B_y)$] in Equation 4a to 0 (yield at the average unexploited equilibrium stock size (B_0) = 0) and solving for K (Eq. 4b).

$$K = HB_0^{m-1} \qquad \textbf{(Eq. 4b)}$$

The yield can now be defined in terms of B_0, H and m (Eq. 4c).

$$g(B_y) = HB_y^m - HB_0^{m-1}B_y \qquad \textbf{(Eq. 4c)}$$

r is redefined as the production rate at B_{msy} (Eq. 4d). B_{msy} can be expressed in terms of B_0 and m (Eq. 4e) by setting the derivative of Equation 4c (with respect to B_y) to zero and solving for B_y (a maximum occurs when the derivative = 0).

$$Y_{msy} = rB_{msy} \qquad \textbf{(Eq. 4d)}$$

$$B_{msy} = \frac{1}{m^{\frac{1}{m-1}}}B_0 \qquad \textbf{(Eq. 4e)}$$

By finding the yield at B_{msy} (Eq. 4f) H can be derived in terms of r, B_0 and m (Eq. 4g). Pella and Tomlinson's model (Eq. 4a) can now be rewritten in terms of the redefined r, m, B_0 (Eq. 5).

$$rB_{msy} = HB_{msy}^m - HB_0^{m-1}B_{msy} \qquad \textbf{(Eq. 4f)}$$

$$H = \frac{r}{\left(\frac{1}{m}-1\right)B_0^{m-1}} \qquad \textbf{(Eq. 4g)}$$

$$g(B_y) = \frac{r}{\left(\frac{1}{m}-1\right)}\left(\frac{B_y^m}{B_0^{m-1}} - B_y\right) \qquad \textbf{(Eq. 5)}$$

where B_0 is the average unexploited equilibrium biomass and r is the production rate at maximum production.

This equation relates the production $[g(B_y)]$ at a particular stock size (B_y) which is a function of the virgin biomass (B_0), current biomass (B_y), the productivity rate at maximum production (r), and a shape parameter (m).

The shape parameter m can be calculated iteratively from Equation 4e to set B_{msy} at any proportion of B_0 (*e.g.*: the Schaefer model can be described by setting $m=2$ which sets B_{msy} at 50% of B_0).

2.2 Stocks Modelled and Fitting Procedure

Three substocks of rock lobster have been identified by the Rock Lobster Working Group: NSN (North NSI), NSC (Central NSI) and NSS (South NSI), as well as a total NSI (North and South Island) stock which is represented by the sum of the three substocks (NST stock; Annala (1993b)). Each of these stocks was modelled from a virgin state using catch histories and catch per unit effort (CPUE) indices of abundance appropriate for each stock (Breen & Kendrick 1994). For every value of B_{msy}/B_0, the r and B_0 were estimated for each stock by fitting predicted biomass to the observed CPUE abundance indices. This was done by relating the two indices using a catchability parameter (q, Eq. 6, assuming the error is additive and normal) and minimising the sum of the squares (Eq. 7) of the weighted differences between the predicted and observed indices. The least squares fitting procedure in this document was used for comparability with an interim rock lobster assessment presented to the Rock Lobster Working Group and differs from the final published assessment (Breen & Kendrick 1994). The catch and CPUE data are given in Breen & Kendrick (1994).

$$I_y = qB_y + \epsilon_y \qquad \textbf{(Eq. 6)}$$

where $\epsilon_y \sim N(0;\sigma^2)$.

$$SS = \sum_{y=1}^{n} \left(w_y \left(I_y - \hat{q}\hat{B}_y \right) \right)^2 \qquad \textbf{(Eq. 7)}$$

Weights were assigned to the CPUE values by year as follows.

	Weight
1945—62	0
1962—73	1
1974—78	0.5
1979—92	1

In an unweighted case, least squares is identical to using maximum likelihood with normal observation error. The weighting has only a small effect on the results (*see* Breen 1991) and these results should be comparable to Breen and Kendrick who used a non-weighted lognormal maximum likelihood.

Because this analysis is used to investigate the sensitivity of parameter estimates to the choice of B_{msy}/B_0, point estimates and confidence intervals were not calculated (*see* Breen & Kendrick (1994) for confidence intervals on the Fox model). Breen & Kendrick predicted the index of abundance assuming it related to mid-year biomass. Their results differ from those presented here as the index of biomass in the current analysis was assumed to occur at the beginning of the year. However, the sensitivity to B_{msy}/B_0 will be the same.

The parameters r, B_0, and q, and the following reference points were estimated for each model:

- Biomass at maximum sustainable yield (B_{msy})
- Maximum sustainable yield (*msy*)
- Current biomass (B_{cur})
- Current surplus production (*CSP*)
- Current biomass as a proportion of virgin biomass (B_{cur}/B_0)
- Current biomass as a proportion of the biomass at maximum sustainable yield (B_{cur}/B_{msy})
- Current surplus production as a proportion of maximum sustainable yield (*CSP/msy*)

These results were compared with the parameter estimates obtained when fitting the Fox model which has B_{msy} fixed at about 37% of B_0. The Fox model used in this analysis (Eq. 8) is different to the formulation used by Breen & Kendrick (1994) (Eq. 9). B_0 is the same but r is the productivity rate at B_{msy} rather than the intrinsic growth rate (r has been redefined for comparability to the r calculated using the P-T model (Eq. 4d)).

$$g(B_y) = rB_y \ln\left(\frac{B_0}{B_y}\right) \qquad \text{(Eq. 8)}$$

$$g(B_y) = rB_y\left(1-\frac{\ln(B_y)}{\ln(B_0)}\right) \qquad \text{(Eq. 9)}$$

3 Results

The ratio B_{cur}/B_0 (Figure 2 and Tables 1 to 4) is not very sensitive to the ratio of B_{msy}/B_0 for all stocks. In the most likely range of the B_{msy}/B_0 ratio (10—50%), B_{cur}/B_0 is constant for NSC, shows a slight increase with B_{msy}/B_0 for NSN, and has a minimum at B_{msy}/B_0=30% and B_{msy}/B_0=45% for NSI and NSS respectively.

B_{cur}/B_{msy} is sensitive to B_{msy}/B_0 (Figure 3) for all stocks and decreases as B_{msy}/B_0 increases.

msy is sensitive to the ratio of B_{msy}/B_0 for all stocks (Figure 4). *msy* increases as the ratio B_{msy}/B_0 decreases for NSN, NSC, and NSI. NSS shows a different pattern with a maximum for *msy* at B_{msy}/B_0=45%.

CSP for the NSS stock is not sensitive to the ratio of B_{msy}/B_0 in the range B_{msy}/B_0=5 to 45%, but it is sensitive to the ratio of B_{msy}/B_0 for the other three stocks (Figure 5). For these stocks, CSP increases as the ratio B_{msy}/B_0 decreases.

CSP as a proportion of *msy* is sensitive to the ratio of B_{msy}/B_0 (Figure 6). Clearly, all stocks have a B_{msy}/B_0 ratio that gives a *CSP* which is equal to *msy*.

r and the B_{msy}/B_0 ratio show a strong relationship in all stocks except the NSS stock (Figure 7) with r increasing as the B_{msy}/B_0 ratio decreases. The NSS stock shows a similar pattern but with a local maximum for r at B_{msy}/B_0=45%. A similar set of plots for B_0 (Figure 8) shows the inverse of the r plots, with B_0 increasing asymptotically at high ratios of B_{msy}/B_0, demonstrating that these two parameters are highly confounded. q (Figure 9) can be considered a nuisance parameter and an examination of all three plots (Figures 7—9) indicates that these parameters are all highly confounded.

A plot of the minimised sum of squares (Figure 10) shows a minimum at about

B_{msy}/B_0=45% for the NSS stock. This corresponds with the maximum for r (Figure 7) and q (Figure 9). No similar minimum can be found for any of the other stocks which all show a continually decreasing sum of squares as the B_{msy}/B_0 ratio reduces.

The results for the Fox model lie on the same predicted line as the Pella-Tomlinson model results (using B_{msy}/B_0=37% in all cases). This indicates that the most important aspect of the Fox model is the location of B_{msy}/B_0, not the algebra of the model formulation.

4 Discussion

In general, the results from the fitted Pella-Tomlinson models are highly sensitive to the value chosen for the B_{msy}/B_0 ratio. This choice clearly has important implications for the reference points which are calculated and used to give management advice. The only reference point that appears to be **not** sensitive to the choice of the B_{msy}/B_0 ratio is the B_{cur}/B_0 ratio. The apparent reason for this is that the B_{cur}/B_0 ratio is constrained by the slope of the CPUE abundance index and the accompanying catch data and is not greatly affected by the choice of model.

It is not clear why the NSS analysis had a local minimum at a B_{msy}/B_0 ratio approximately equal to 45%. This is probably due to unexplored properties of the data specific to this substock.

It would appear that choosing a B_{msy}/B_0 ratio requires considerations similar to those used when choosing the stock recruitment parameters in a fully age-structured model. In those cases, it is customary to choose these parameters with reference to known biological characteristics of the stock being considered. Similarly, the choice of a B_{msy}/B_0 ratio should also be made on the basis of stock characteristics.

There are substantial differences in population dynamics between the N.Z. rock lobster substocks which have been recognised by the Rock Lobster Working Group. These differences arise mainly from differences in growth rates, in the size of animals at the onset of maturity, and in the minimum legal size which is used in each area. Such differences have important implications when choosing a ratio for B_{msy}/B_0 and consequently for the overall productivity of the stock. It may be more reasonable to use a higher ratio of B_{msy}/B_0 when the entire spawning stock is vulnerable to the fishery. On the other hand, lower ratios of B_{msy}/B_0 can be used when a substantial fraction of the spawning population is protected from harvest (through, for instance, a minimum size limit).

It is probably not correct to apply a single B_{msy}/B_0 ratio to all the N.Z. rock lobster substocks without first establishing the basis for the choice. Either an estimate of the P-T shape parameter (m) should be made based on the available data or reasoning external to the model is brought forward to justify the choice. Unfortunately, the final result of the stock assessment can be largely predetermined by the choice of B_{msy}/B_0 ratio.

5 References

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Table 1. Results of analysis for North-NSI rock lobster (NSN) stock. Values in parentheses are negative and have no biological meaning.

Model	B_{cur}/B_0	B_{cur}/B_{msy}	B_{cur} (t)	*CSP* (t)	B_{msy} (t)	*msy* (t)	*r*	B_0 (t)	*q*	SS	*CSP/msy*
2.5%B_0	37.2%	1524.2%	3 452	508	226	743	3.2814	9 270	1.3296e-04	0.1805	68.3%
5%B_0	37.3%	759.8%	3 458	506	455	699	1.5364	9 271	1.3246e-04	0.1812	72.3%
10%B_0	37.5%	374.7%	3 478	501	928	632	0.6805	9 287	1.3106e-04	0.1827	79.4%
15%B_0	37.6%	251.8%	3 508	495	1 393	581	0.4172	9 320	1.2922e-04	0.1847	85.2%
20%B_0	37.9%	190.0%	3 552	488	1 869	540	0.2891	9 377	1.2678e-04	0.1871	90.2%
25%B_0	38.3%	153.4%	3 630	477	2 367	506	0.2137	9 474	1.2357e-04	0.1900	94.3%
30%B_0	38.5%	128.5%	3 708	464	2 886	476	0.1650	9 623	1.1954e-04	0.1936	97.5%
37%B_0 Fox	39.1%	106.3%	3 892	441	3 660	442	0.1207	9 948	1.1249e-04	0.1994	99.8%
40%B_0	39.4%	98.8%	4 008	427	4 056	427	0.1053	10 164	1.0858e-04	0.2026	100.0%
50%B_0 Schaefer	40.6%	81.3%	4 603	368	5 664	381	0.0674	11 327	9.2795e-05	0.2145	96.5%
60%B_0	42.1%	70.2%	5 919	280	8 435	322	0.0382	14 060	7.1185e-05	0.2280	86.9%
70%B_0	43.9%	62.7%	12 803	59	20 428	79	0.0039	29 187	3.3244e-05	0.2380	73.9%
80%B_0	44.6%	55.8%	62 185	(1 112)	111 433	(1 828)	-0.0164	139 389	7.0610e-06	0.2294	60.8%
90%B_0	41.7%	46.4%	347 258	(7 490)	748 832	(15 680)	-0.0209	832 618	1.2936e-06	0.2224	47.8%

Table 2. Results of analysis for Central-NSI rock lobster (NSC) stock. Values in parentheses are negative and have no biological meaning.

Model	B_{cur}/B_0	B_{cur}/B_{msy}	B_{cur} (t)	*CSP* (t)	B_{msy} (t)	*msy* (t)	*r*	B_0 (t)	*q*	SS	*CSP/msy*
2.5%B_0	32.8%	1341.0%	11 699	1 747	872	2 392	2.7418	35 708	4.1911e-05	0.5901	73.0%
5%B_0	32.7%	666.9%	11 716	1 734	1 757	2 245	1.2781	35 786	4.1699e-05	0.5905	77.2%
10%B_0	32.7%	328.4%	11 765	1 701	3 582	2 017	0.5631	35 992	4.1174e-05	0.5918	84.3%
20%B_0	32.6%	163.4%	11 940	1 606	7 308	1 696	0.2321	36 663	3.9710e-05	0.5953	94.7%
25%B_0	32.1%	128.5%	11 952	1 541	9 299	1 569	0.1687	37 203	3.8704e-05	0.5979	98.2%
30%B_0	32.4%	108.2%	12 307	1 454	11 374	1 458	0.1282	37 927	3.7503e-05	0.6010	99.8%
37%B_0 Fox	32.4%	88.1%	12 745	1 312	14 472	1 321	0.0913	39 339	3.5496e-05	0.6063	99.3%
40%B_0	32.4%	81.2%	13 022	1 234	16 044	1 262	0.0787	40 199	3.4426e-05	0.6092	97.8%
50%B_0 Schaefer	32.5%	65.1%	14 392	945	22 120	1 076	0.0486	44 239	3.0362e-05	0.6205	87.8%
60%B_0	33.1%	55.1%	16 844	640	30 564	882	0.0288	50 942	2.5653e-05	0.6339	72.6%
70%B_0	34.2%	48.9%	21 779	363	44 574	622	0.0140	63 684	2.0173e-05	0.6476	58.3%
80%B_0	35.4%	44.2%	39 631	(115)	89 585	(239)	-0.0027	112 060	1.1896e-05	0.6536	48.2%
90%B_0	34.5%	38.4%	48 012	(317)	125 010	(802)	-0.0064	138 997	1.0030e-05	0.6515	39.6%

Table 3. Results of analysis for South-NSI rock lobster (NSS) stock. Values in parentheses are negative and have no biological meaning.

Model	B_{cur}/B_0	B_{cur}/B_{msy}	B_{cur} (t)	*CSP* (t)	B_{msy} (t)	*msy* (t)	*r*	B_0 (t)	*q*	SS	*CSP/msy*
$5\%B_0$	20.0%	400.4%	15 827	1 354	3 953	1 502	0.3800	79 312	6.1047e-05	0.7770	90.1%
$10\%B_0$	19.8%	196.0%	15 357	1 358	7 834	1 416	0.1807	77 617	6.2924e-05	0.7724	95.9%
$15\%B_0$	19.6%	130.9%	14 790	1 362	11 295	1 375	0.1217	75 549	6.5322e-05	0.7670	99.1%
$20\%B_0$	19.3%	96.8%	14 043	1 368	14 508	1 369	0.0943	72 764	6.8783e-05	0.7602	100.0%
$25\%B_0$	18.9%	75.8%	13 062	1 378	17 228	1 401	0.0813	68 992	7.3949e-05	0.7516	98.3%
$30\%B_0$	18.4%	61.5%	11 762	1 393	19 130	1 484	0.0776	63 790	8.2166e-05	0.7408	93.8%
$35\%B_0$	17.7%	50.7%	9 974	1 419	19 674	1 645	0.0836	56 215	9.7143e-05	0.7269	86.3%
$37\%B_0$ Fox	17.0%	46.1%	9 152	1 434	19 335	1 733	0.0896	52 557	1.0610e-04	0.7207	82.7%
$40\%B_0$	16.6%	41.7%	7 307	1 477	17 532	1 959	0.1117	43 910	1.3406e-04	0.7070	75.4%
$45\%B_0$	15.3%	33.9%	4 550	1 471	13 413	2 387	0.1779	29 807	2.1066e-04	0.6801	61.7%
$50\%B_0$ Schaefer	15.3%	30.7%	5 030	1 215	16 405	2 340	0.1426	32 809	1.6826e-04	0.7566	51.9%
$60\%B_0$	17.0%	28.4%	9 901	698	34 909	1 762	0.0505	58 185	7.3346e-05	0.9986	39.6%
$70\%B_0$	20.1%	28.7%	39 130	(642)	136 207	(1 866)	-0.0137	194 605	1.8336e-05	1.0375	34.4%
$80\%B_0$	21.2%	26.5%	235 030	(8 995)	885 905	(31 115)	-0.0351	1,108,162	3.1178e-06	0.9404	28.9%
$90\%B_0$	17.3%	19.2%	400 185	(15 913)	2,078,941	(80 260)	-0.0386	2,311,548	1.8446e-06	0.9196	19.8%

Table 4. Results of analysis for North-South Island rock lobster (NSI) stock. Values in parentheses are negative and have no biological meaning.

Model	B_{cur}/B_0	B_{cur}/B_{msy}	B_{cur} (t)	*CSP* (t)	B_{msy} (t)	*msy* (t)	*r*	B_0 (t)	*q*	SS	*CSP/msy*
2.5%B_0	17.0%	656.2%	11 078	4 100	1 688	4 594	2.7215	65 171	6.6601e-05	0.1826	89.2%
5%B_0	16.8%	338.7%	10 801	4 093	3 189	4 405	1.3815	64 202	6.7245e-05	0.1842	92.9%
10%B_0	16.4%	164.6%	10 286	4 058	6 250	4 143	0.6629	62 553	6.7323e-05	0.1942	97.9%
20%B_0	15.9%	79.7%	10 044	3 814	12 599	3 846	0.3052	63 205	5.9540e-05	0.2541	99.2%
25%B_0	15.8%	63.3%	10 474	3 565	16 560	3 720	0.2247	66 282	5.2517e-05	0.3044	95.8%
30%B_0	15.9%	52.9%	11 352	3 232	21 441	3 581	0.1670	71 496	4.4911e-05	0.3614	90.3%
37%B_0 Fox	16.3%	44.3%	13 518	2 678	30 512	3 331	0.1092	82 940	3.4799e-05	0.4408	80.4%
40%B_0	16.6%	41.7%	15 034	2 397	36 080	3 179	0.0881	90 403	3.0493e-05	0.4752	75.4%
50%B_0 Schaefer	18.3%	36.7%	23 749	1 430	64 759	2 387	0.0369	129 518	1.8711e-05	0.5657	59.9%
60%B_0	20.8%	34.7%	44 674	262	128 899	545	0.0042	214 843	1.0321e-05	0.6079	48.0%
70%B_0	23.2%	33.1%	113 054	(2 462)	341 697	(6 213)	-0.0182	488 199	4.3859e-06	0.5809	39.6%
80%B_0	24.2%	30.3%	641 380	(22 828)	2e+06	(69 230)	-0.0327	3e+06	8.2906e-07	0.5039	33.0%
90%B_0	20.5%	22.7%	1e+06	(53 524)	6e+06	(2e+05)	-0.0357	7e+06	3.7446e-07	0.4894	23.4%

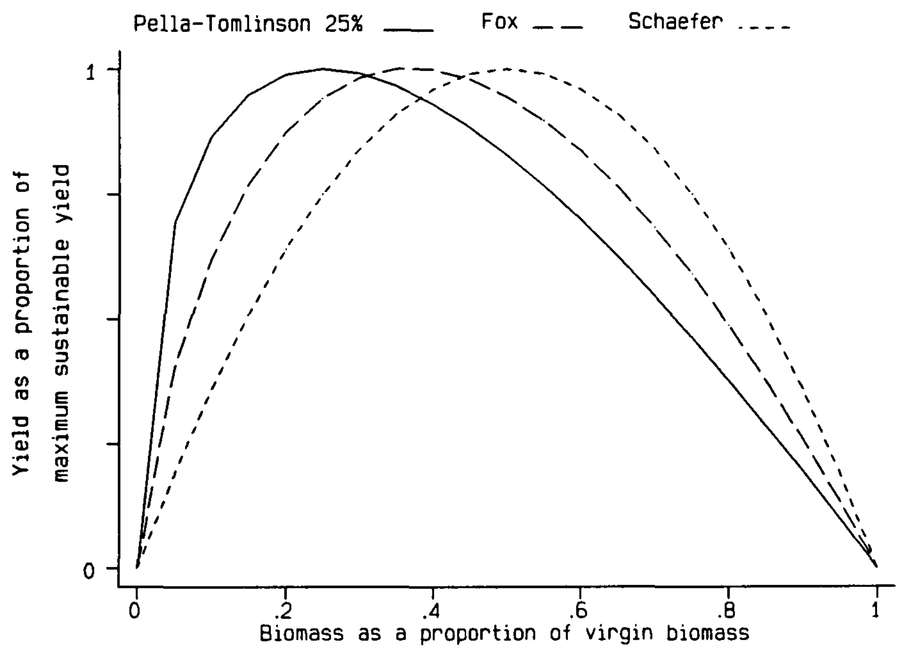

Figure 1. Relationship between surplus production and biomass for Fox, Schaefer, and Pella-Tomlinson (with B_{msy} at 25% of B_0) models.

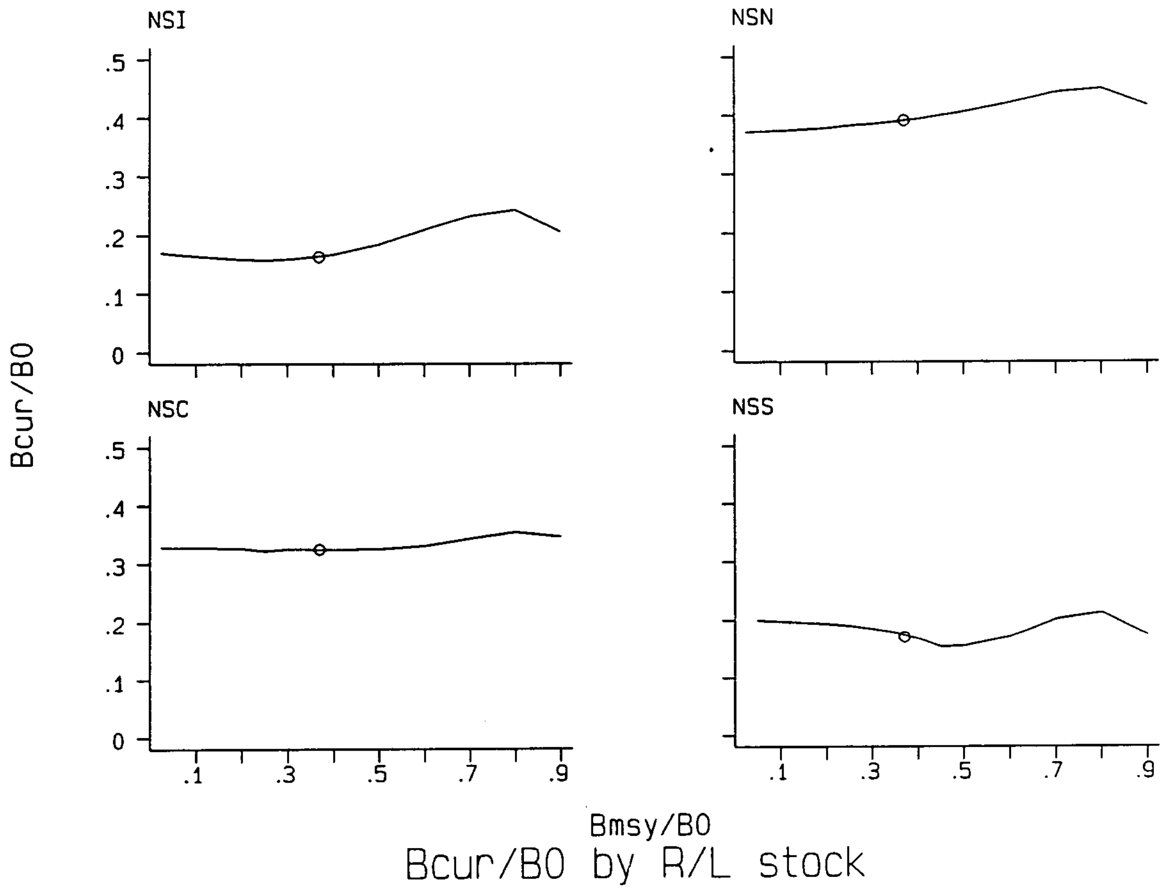

Figure 2. The estimated ratio of current biomass to virgin biomass at different levels of B_{msy}/B_0 using the Pella-Tomlinson model for the four stocks. "o" represents the Fox model estimates.

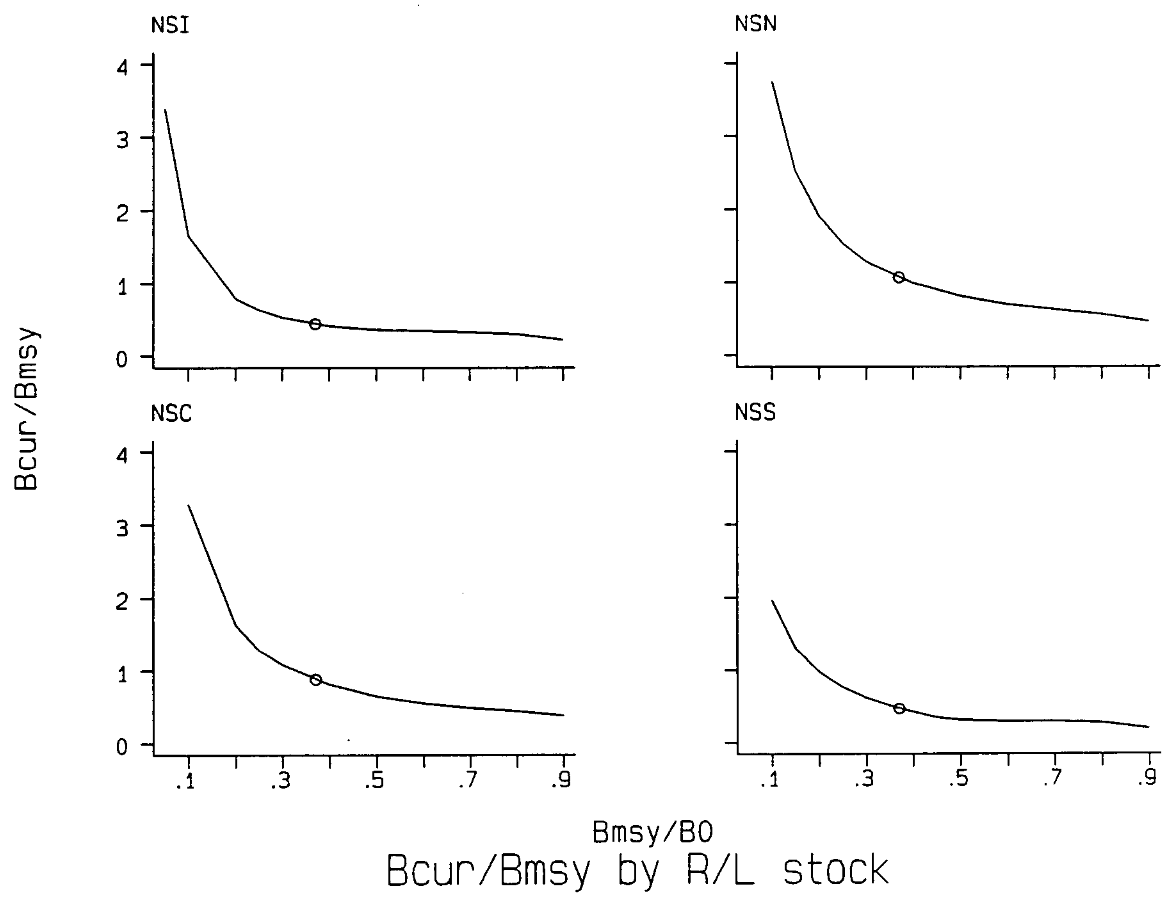

Figure 3. The estimated ratio of current biomass to B_{msy} at different levels of B_{msy}/B_0 using the Pella-Tomlinson model for the four stocks. "o" represents the Fox model estimates.

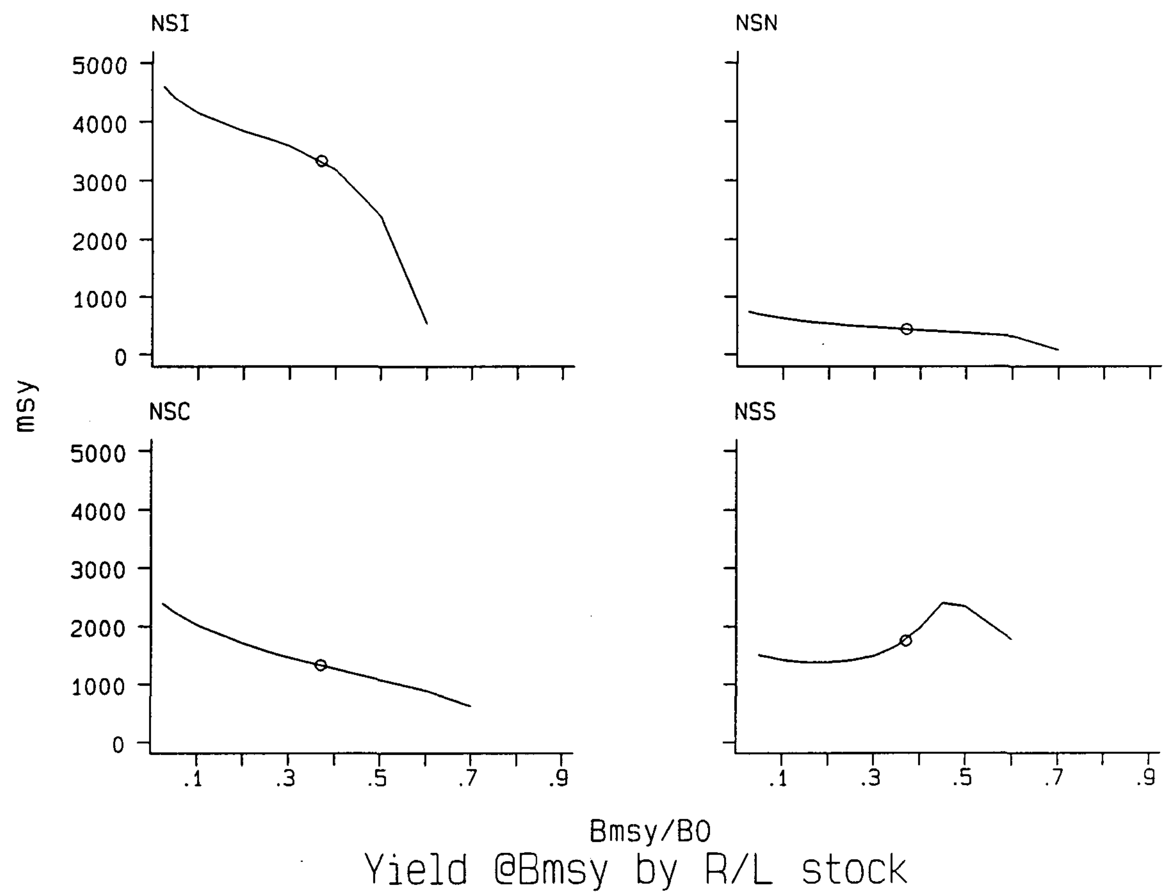

Figure 4. Estimated *msy* (t) at different levels of B_{msy}/B_0 using the Pella-Tomlinson model for the four stocks. "o" represents the Fox model estimates.

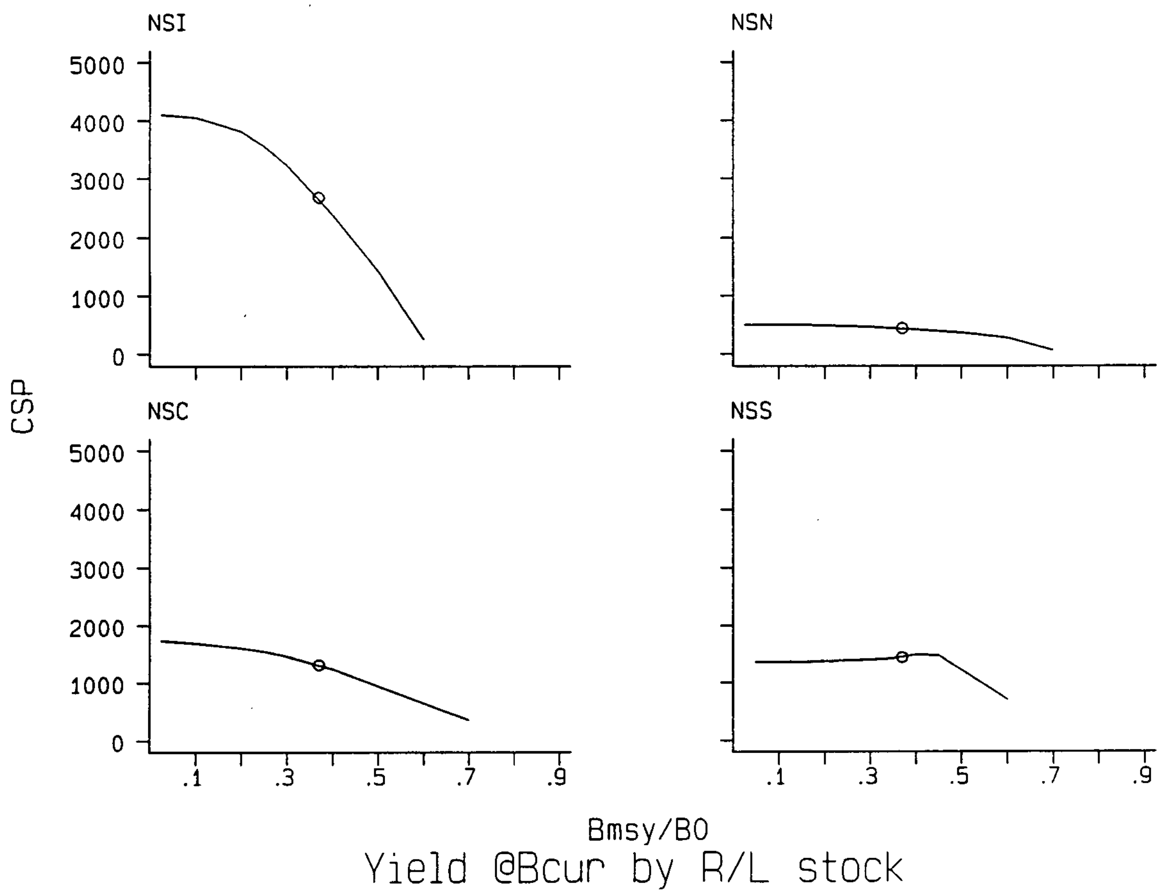

Figure 5. Estimated current surplus production (t) at different levels of B_{msy}/B_0 using the Pella-Tomlinson model for the four stocks. "o" represents the Fox model estimates.

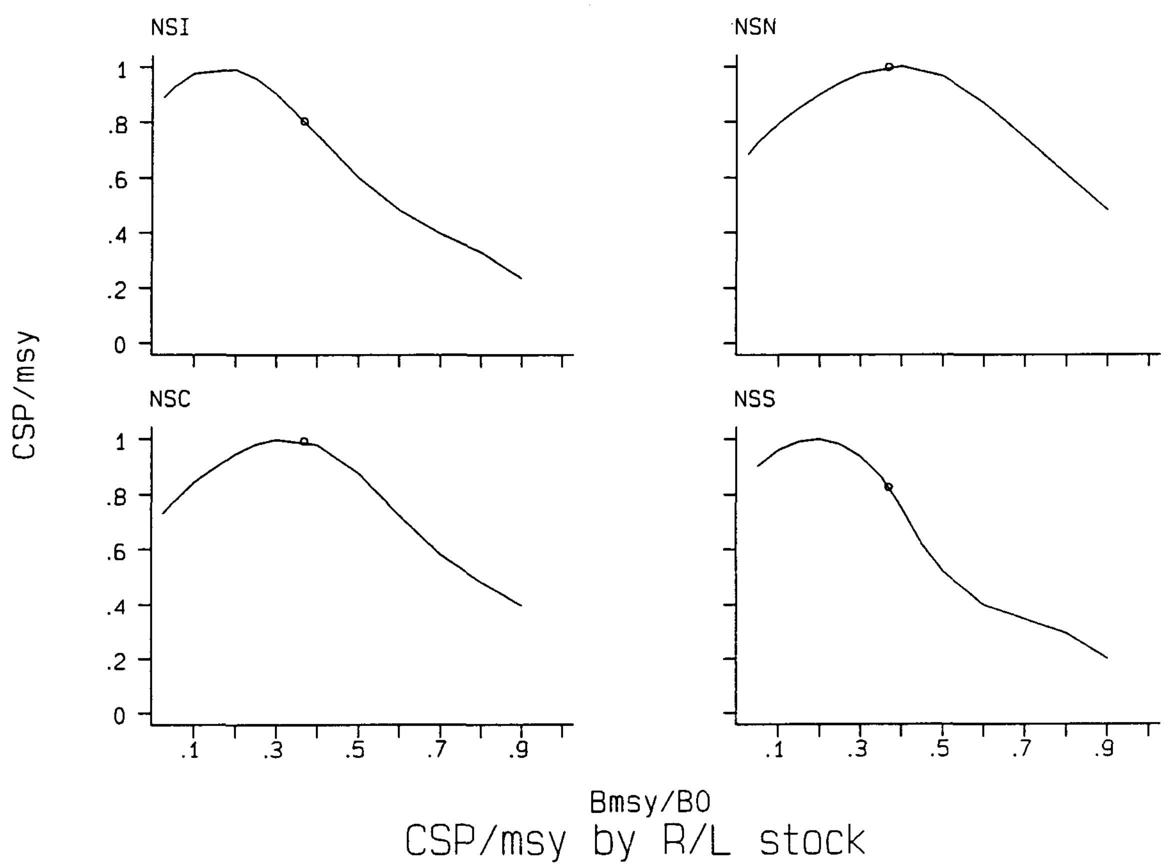

Figure 6. Current surplus production as a proportion of maximum sustainable yield at different levels of B_{msy}/B_0 for the four stocks. "o" represents the Fox model estimates.

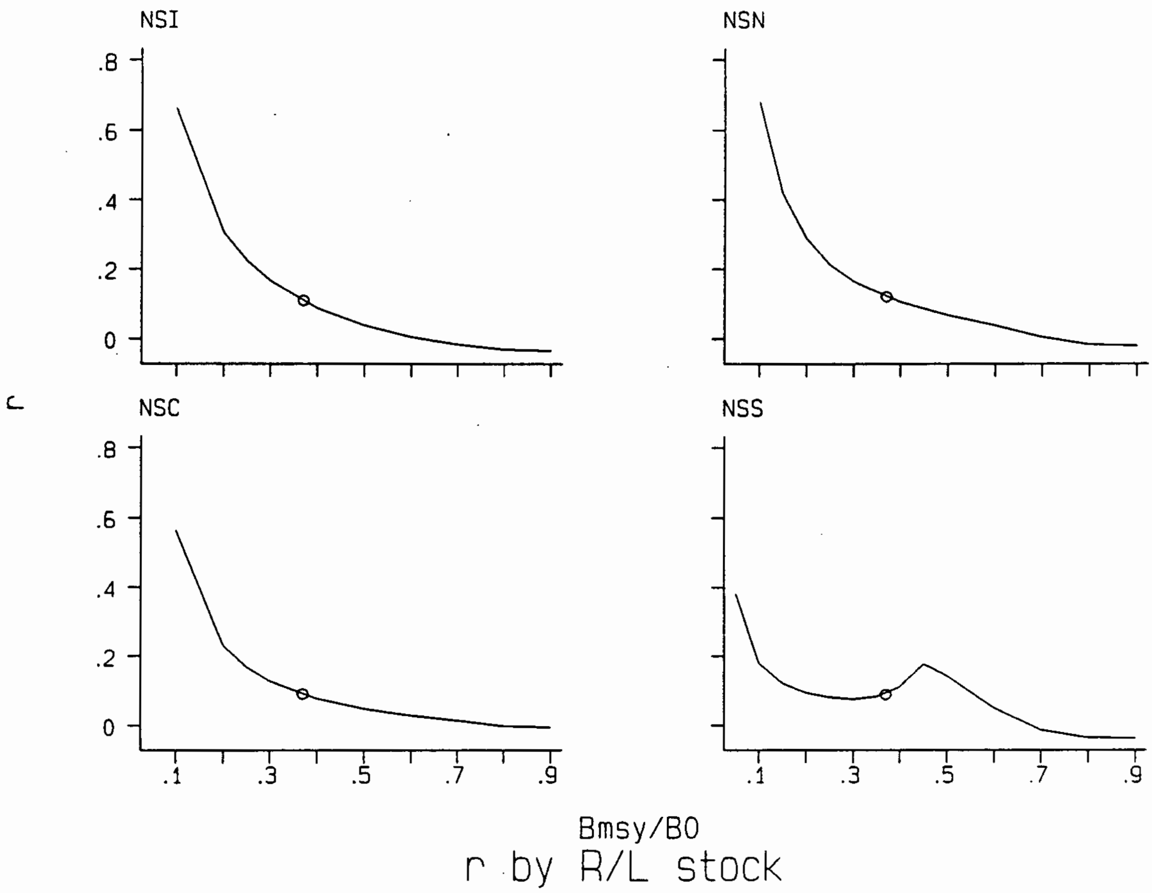

Figure 7. The estimated productivity parameter, *r*, at different levels of B_{msy}/B_0 using the Pella-Tomlinson model for the four stocks. "o" represents the Fox model estimates.

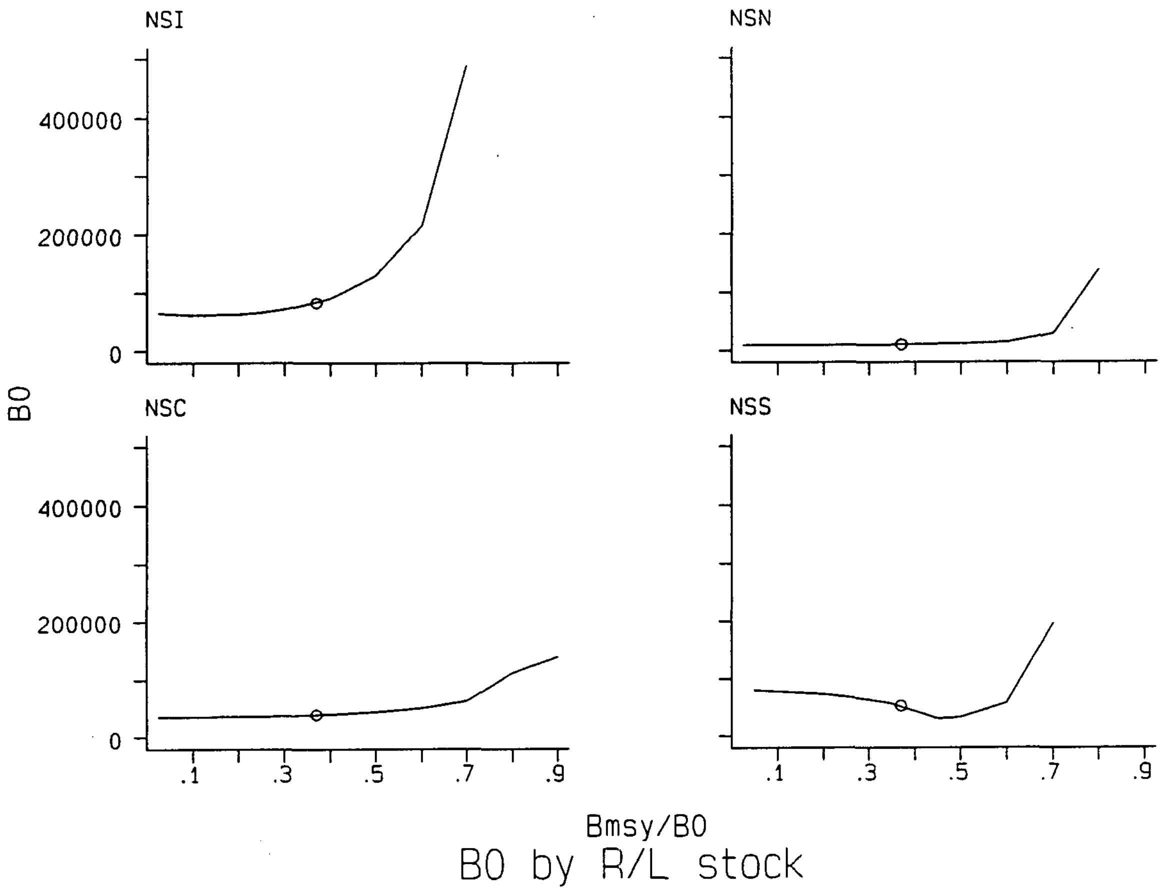

Figure 8. The estimated virgin biomass (B_0 [t]) at different levels of B_{msy}/B_0 for the four stocks. "o" represents the Fox model estimates.

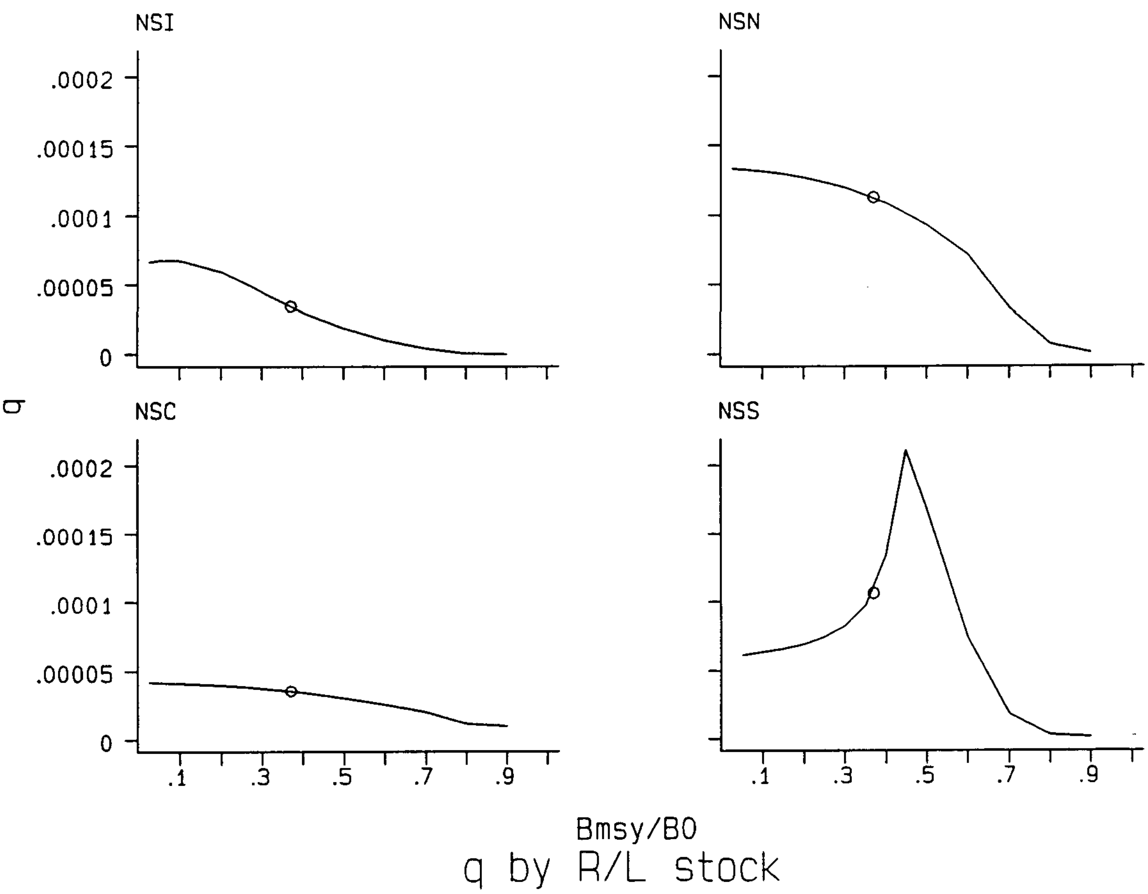

Figure 9. The estimated catchability coefficient (q) at different levels of B_{msy}/B_0 using the Pella-Tomlinson model for the four stocks. "o" represents the Fox model estimates.

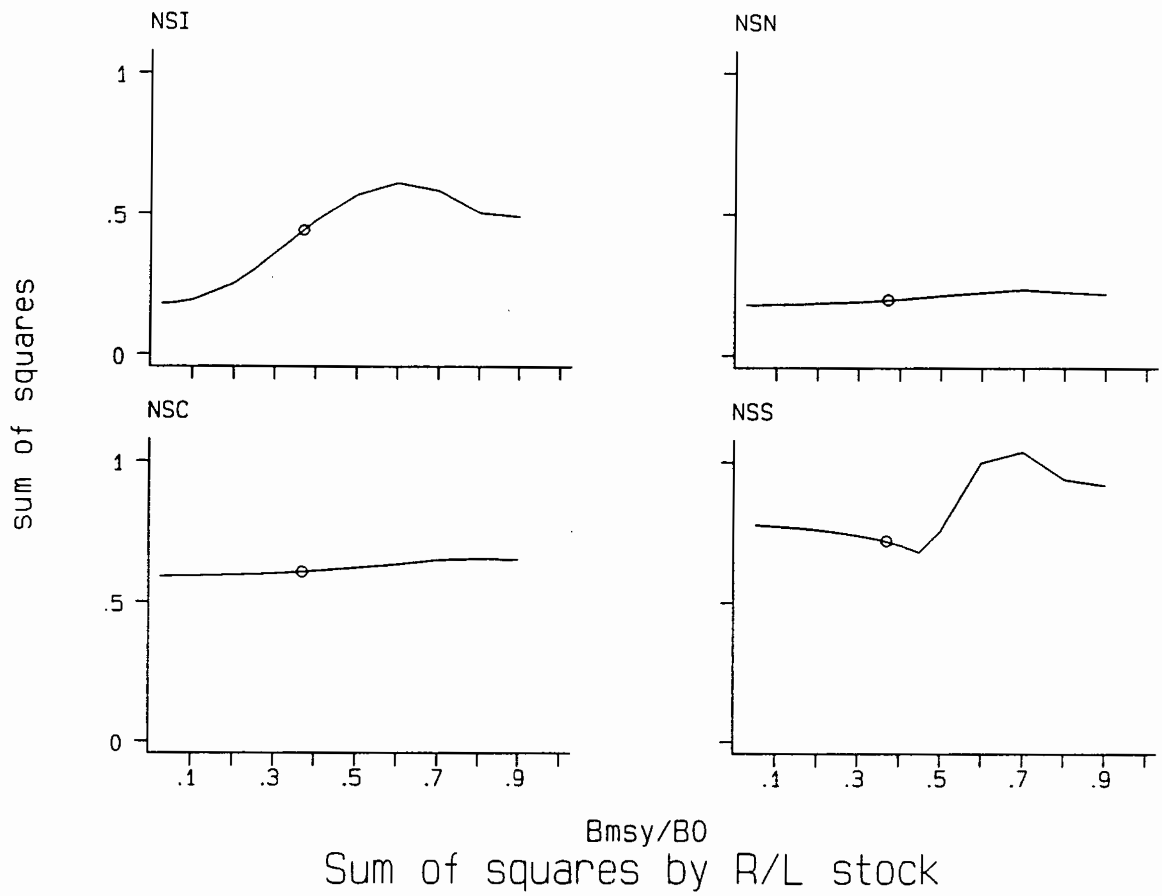

Figure 10. The sums of squares at different levels of B_{msy}/B_0 using the Pella-Tomlinson model for the four stocks. "o" represents the Fox model estimates.