HANDBOOK OF HYDROLOGICAL PROCEDURES:
PROCEDURE No. 8

BASE-FLOW-RECESSION CURVES

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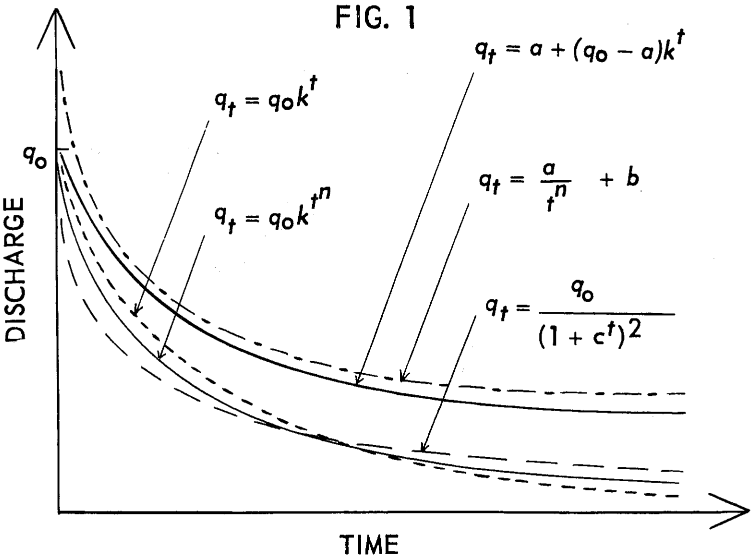
1. Definition

The base-flow-recession curve is the lower part of the falling limb of a hydrograph and expresses the relation between base flow and time. Since base flow is largely ground-water effluent, the base-flow-recession curve represents mainly withdrawal from ground-water storage. However – depending on the water-table level, the river-bank material, the soils in the neighbourhood of the river bank, and the rates of rise and fall of the stream – withdrawal from bank storage will contribute to base flow.

2. Recession-curve equations

Several curves have been suggested as representing base-flow recession. The most important of these are: simple exponential, double exponential, and hyperbolic. In conditions where ice melt contributes to the base flow, the ice-melt exponential and the ice-melt hyperbola can be used. (See fig. 1).

FIG. 1



2.1 *Simple Exponential:*

The withdrawal, assuming no inflow from precipitation, is an exhaustion process giving a base-flow-recession curve of the form:

$$q_t = q_o e^{-at}$$

$$= q_o k^t \qquad \ldots\ldots \quad (1)$$

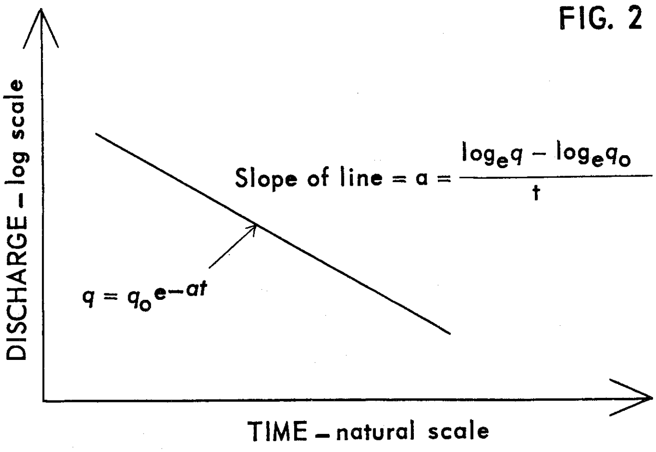
where q_o = initial discharge,

q_t = discharge at time t,

$k = e^{-a}$ = recession constant.

This curve, plotted on semi-log graph paper, is represented as a straight line with slope $-a$. (See fig. 2).

FIG. 2



2.2 *Double Exponential:*

On constructing a base-flow recession, the curve plotted on semi-log graph paper is often found to be nonlinear. The double exponential can be made to fit the data empirically, with the form:

$$q_t = q_o e^{-bt^n} \quad \ldots\ldots \quad (2)$$

where $e^{-b} = k$, the recession constant.
On taking natural logarithms (twice) of both sides:

$$\ln\left(\frac{q_o}{q_t}\right) = bt^n$$

$$\therefore \quad \ln\left[\ln\left(\frac{q_o}{q_t}\right)\right] = n\ln t + \ln b$$

$$\therefore \quad \log\left[\log\left(\frac{q_o}{q_t}\right)\right] = n\log t + \log b - 0.36222.$$

The constants are derived by the method of least squares.

2.3 *Hyperbola:*

Another curve that has been suggested as a fit of the recession is the hyperbola of the form:

$$q_t = \frac{q_o}{(1 + ct)^2} \quad \ldots\ldots \quad (3)$$

or, taking logs:

$$\log q_t = \log q_o - 2\log(1 + ct).$$

The parameter c can be calculated directly from q_o, q_t and t as

$$c = \frac{1}{t}\left(\sqrt{\frac{q_o}{q_t}} - 1\right)$$

This is usually just a theoretical recession curve, but it will occur if ground-water storage is at its maximum after the cessation of surface run-off, or if no further precipitation or snow melt occurred until stream flow ceased. For humid climates, where there are only short periods of dry weather, these conditions rarely occur.

2.4 *Ice-melt hyperbola:*

Where ice and snow are present the recession constant approaches unity. An equation is used of the form:

$$q = \frac{a}{t^n} + b \qquad \ldots. \quad (4)$$

where a and b are constants. Because this curve asymptotically approaches a constant discharge it may typify base-flow recessions in areas where permanent snow and ice are present; for this reason it is called an ice-melt hyperbola. Since for this equation:

$$\log(q_t - b) = -n\log t + \log a,$$

a plot on log-log paper of $(q_t - b)$ and t will result in a straight line of slope $-n$, provided that b has been chosen correctly. A study of the stream flow data should give an approximate estimate of a constant b for initial trials.

2.5 *Ice-melt Exponential:*

A curve-fitting equation suitable for a sustained slow recession rate is of the form:

$$q_t = a + (q_o - a)k^t \qquad \ldots. \quad (5)$$

where a and k are constants. This equation is similar to equation (4) in that for large values of t the discharge asymptotically approaches a constant value. Since in many cases the discharge, which is largely supplied by melt from permanent ice and snowfields, reaches an apparently constant value for some finite duration, then this equation, like the ice-melt hyperbola, is suitable for catchments that are strongly influenced by snow and ice – provided one is cautious not to extend such curves too far in time. However, unlike equation (4) the curve is relatively flat for small values of t. The logarithmic form of equation (5) is:

$$\log(q_t - a) = t\log k + \log(q_o - a)$$

hence for a suitable constant a, a plot of $(q_t - a)$ on log scale against t (natural scale) will produce a straight line of slope $\log k$.

3. Master base-flow-recession curve

A composite recession curve compiled for the largest range of base-flow discharge values obtainable is called the master base-flow-recession curve. A master base-flow-recession curve was drawn from 1964 flow data from Puketurua experimental basin. The portion of this recession curve from 2 cusecs to 0.2 cusec was found to plot as a straight line on semi-log graph paper. Two values, 18 days apart, were taken from this curve to calculate the recession constant k,

at $t = 0$ days, $q = 1.59$ cusecs;
at $t = 18$ days, $q = 0.201$ cusec.

Thus, substituting these values in equation (1),

$$0.201 = 1.59\, k^{18}$$

$$\log k = \frac{\log 0.201 - \log 1.59}{18}$$

$$= \bar{1}.9501$$

$$k = 0.8915$$

$$\therefore q_t = q_o 0.892^t$$

Master base-flow-recession curves tend to deviate from a straight line when plotted on semi-log paper, the value of *k* increasing with time.

4. Construction of master recession curve

A recession curve can be constructed by:

- (i) carrying out a series of flow measurements;
- (ii) the strip method;
- (iii) the correlation method;
- (iv) the tabulation method.

Method (i) gives a single recession curve, valuable for prediction purposes when correlated with rainfall records, but the curve is not described as a master recession curve.

5. Construction using flow measurements

A series of current-meter gaugings should be carried out at approximately two-day intervals and extended over a complete period of dry weather.

The gaugings should be as accurate as possible and the gauge height should be recorded with each gauging to ±0.005 ft.
The gaugings should be plotted against time on semi-log paper and connected by a straight line or smooth curve. (See fig. 3).

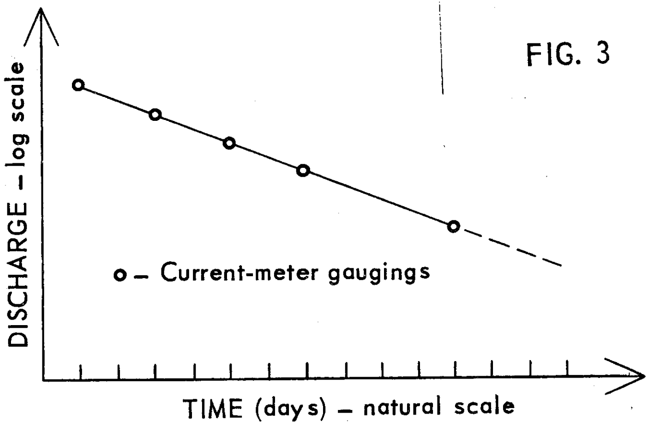


FIG. 3

6. Construction using the strip method

Recessions occurring during periods of dry weather are extracted from the stage records (see fig. 4 for a chart record).

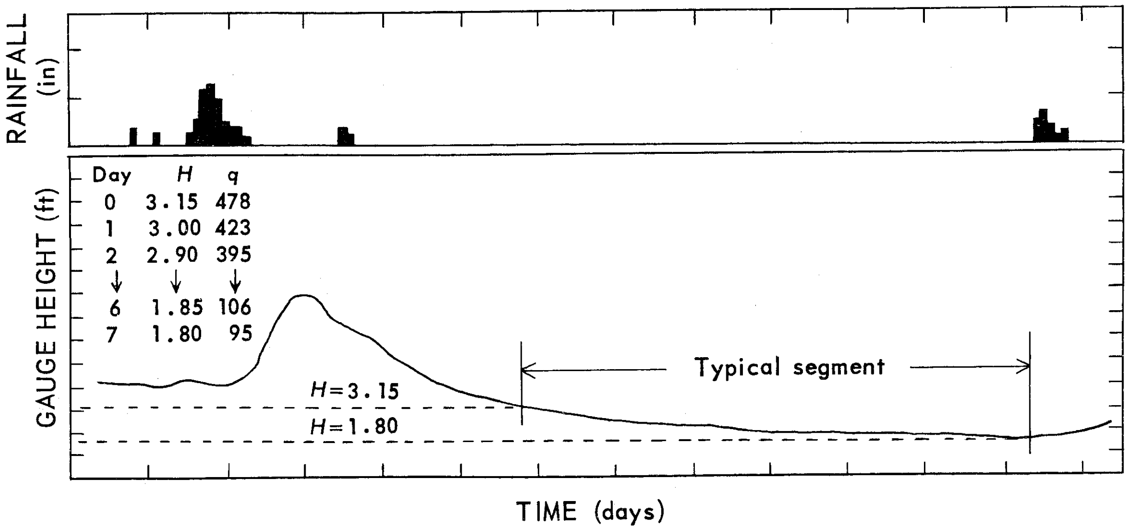


FIG. 4: Rainfall chart and recorder chart.

When choosing recession periods, allow sufficient time between the time chosen for the beginning of the period and the time of the last recorded rainfall so that resulting surface flow will have passed the gauging station.

If the portion of the recession resulting from surface flow is not readily identifiable, then the lag may be estimated, and any surface flow tabulated will become apparent in the construction of the master recession curve.

6.1 The recessions chosen should persist for periods of at least 7 days. For each recession, list gauge heights at daily intervals (or some other regular period) and transform to discharge by means of the station rating table.

6.2 Plot the discharge recessions on semi-log paper. Break up the abscissae into arbitrary time scales so as to prevent overcrowding of curves. (See fig. 5).

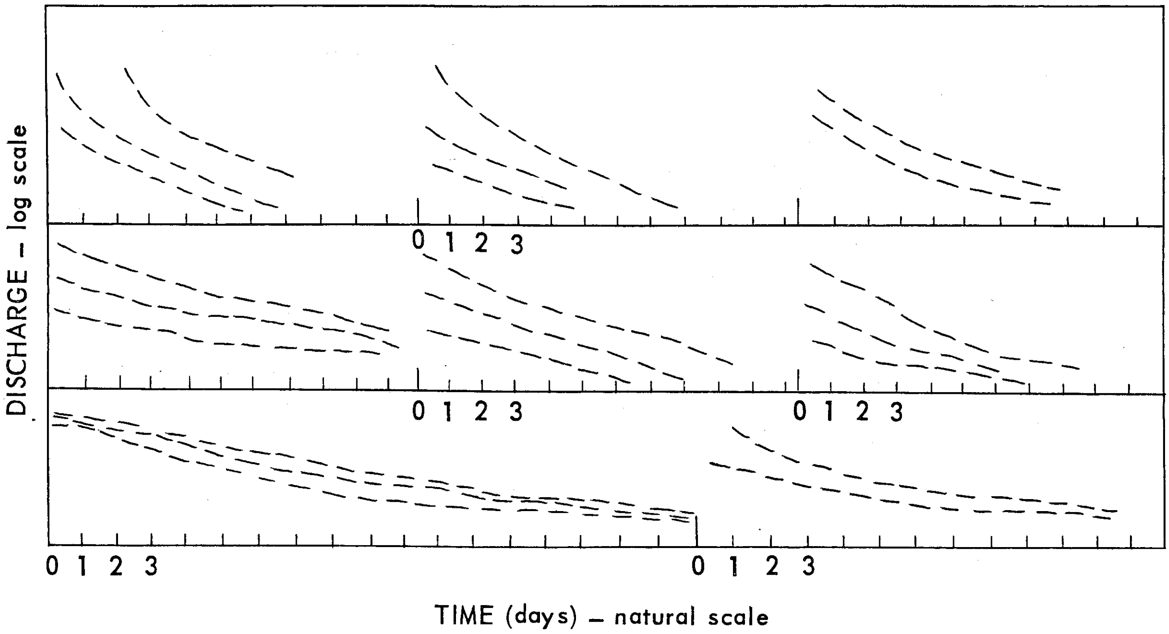


FIG. 5. Segments of recession curve

Identify each curve – a reference to the complete record may be necessary at a later stage.

Recessions within the same discharge range will have sensibly equal slopes.

Surface flow may be identified by an excessive rate of recession; snow melt or unrecorded rainfall by a decreased rate of recession.

Seasonal recessions – winter and summer curves – are in some cases identified by their slopes.

6.3 The segments of discharge are combined by plotting individual recessions on tracing paper; they are then superimposed and adjusted horizontally until the main parts overlap. A mean line or curve through the overlapping parts is the master recession curve. (See fig. 6).

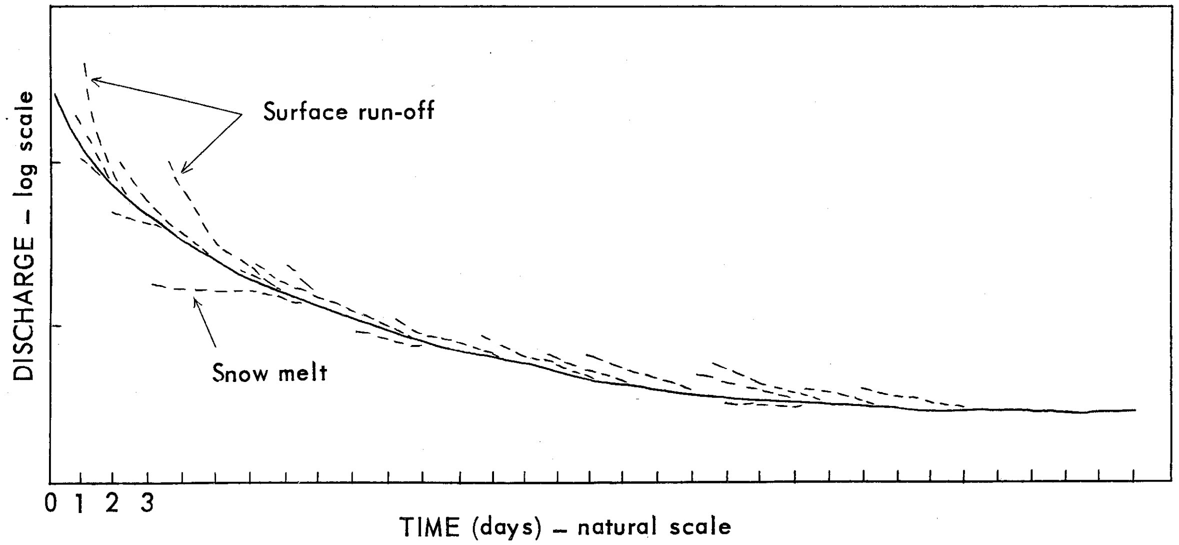


FIG. 6: Complete recession curve

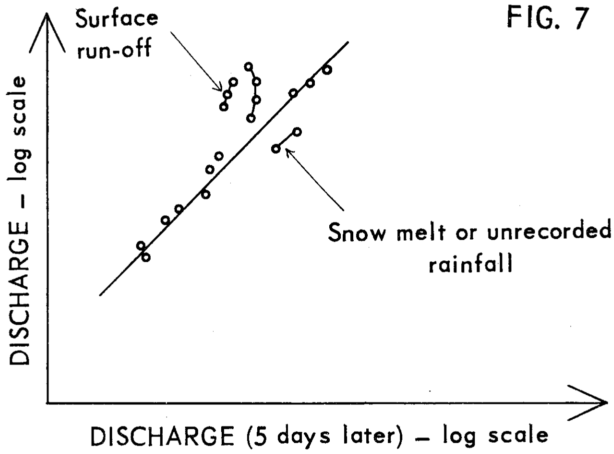
The resulting curve indicates the minimum base flow for the period of the record; however, this curve is not necessarily reproducible for a subsequent period of corresponding length, as aquifer storage is not a constant but depends on antecedent recharge rates.

7. Construction using the correlation method

Hydrograph segments are derived as explained in section 6.

Plot on log-log paper the discharge for the initial day of each segment, starting with the highest discharge, with a discharge c days later (where c = ½, 1, 2, 5, etc.) as the abscissa, and continue for as many points as the segment length permits. This procedure is followed for as many segments as possible until a good definition of the correlation is obtained. (See fig. 7).

FIG. 7

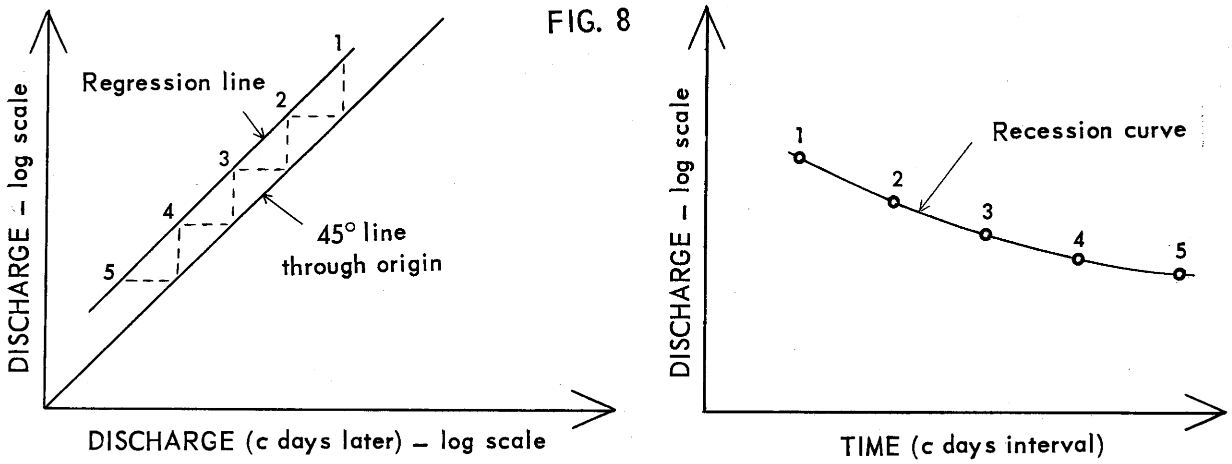


The points are averaged by a straight line or by a smooth curve (the regression line of $\log q$ on $\log q_c$ could be fitted).

Points which lie above the mean line indicate surface flow (rapid recession); points below the line show a slower rate of recession and could be due to snow melt or unrecorded rainfall. The master base-flow-recession curve may be constructed from fig. 7 by taking successive steps from the largest discharge to the smallest.

For this purpose a 45-degree line is drawn through the origin and successive values read off the ordinate as shown in fig. 8.

FIG. 8



8. Construction using the tabulation method

This method is essentially the same as the method described in section 6, except that tabulated data are used in place of graphical data.

For this method, either daily mean discharges or instantaneous discharges at regular intervals of time (e.g. every 12 hours) are tabulated in vertical columns, one column for each recession. (The segments of the hydrograph are chosen as in sections 6 and 7.)

The columns are adjusted vertically until the discharges approximately agree horizontally. Subsequently the discharges are averaged horizontally, and these mean discharges constitute the master recession curve. This method gives good control of the data, making it less probable that the final curve will be too long or too short. The disadvantage of this method is that seemingly irrelevant parts of the recession cannot be omitted without a detailed inspection.

Six segments were chosen from hydrographs from the Puketurua 1964 flow data. (These were listed with instantaneous flows at 12-hour intervals and then aligned and averaged. The commencing times are listed for reference. All values are in cusecs.

3 Aug 1200hrs	*6 Sep 0000hrs*	*19 Jul 0000hrs*	*5 Jun 0000hrs*	*3 Nov 1200hrs*	*24 Dec 0000hrs*	*Average Discharge (cusecs)*
2.431						2.431
2.025						2.025
1.917						1.917
1.882						1.882
1.632						1.632
1.567						1.567
1.520	1.520					1.520
1.429	1.429	1.474				1.444
1.342	1.342	1.342				1.342
1.299	1.258	1.258				1.272
1.217	1.138	1.217				1.191
1.138	1.099	1.138				1.125
1.099	1.062	1.099				1.087
1.025	1.025	1.025				1.025
	0.989	0.989				0.989
	0.954	0.920				0.937
	0.820	0.885				0.852
	0.788		0.820			0.804
	0.758		0.758			0.758
	0.698		0.698			0.698
	0.670		0.642			0.656
	0.642		0.615			0.628
	0.615		0.588			0.601
	0.562		0.562			0.562
	0.537		0.513			0.525
	0.513		0.489	0.513		0.501
			0.466	0.466	0.489	0.474
			0.422	0.422	0.444	0.429
			0.401	0.380	0.401	0.394
				0.361	0.361	0.361
				0.341	0.323	0.331
				0.305	0.305	0.305
				0.288		0.288

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