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Handbook on Mixing in Rivers

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HANDBOOK ON MIXING IN RIVERS

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ABSTRACT

This handbook briefly describes the mechanisms of solute mixing in rivers and gives equations for these processes. Using these equations, with worked examples, simple techniques are given for predicting rates of mixing in rivers. The problems dealt with are those which can be solved conveniently using nomographs, programmable calculators or, at most, a small mini-computer. Use of the semi-empirical techniques described can provide a preliminary assessment of the impact of effluent on water quality.

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LIST OF SYMBOLS

a	concentration ratio C_p/C
A	cross-sectional area
A_z, A_y	major and minor axes of concentration contours, see Figures 5.1, 5.2
b	channel width
C	concentration
$\bar{C}$	fully mixed concentration
C^*	non-dimensional concentration $C/\bar{C}$
C_p	peak concentration
C_m	minimum detectable concentration
D	molecular diffusion coefficient
D_x, D_y, D_z	dispersion coefficient in x, y and z directions
D_o, D_s	dispersion coefficient in unstratified and stratified flow
d	channel mean depth
E_x, E_y, E_z	turbulent diffusion coefficient in x, y and z directions
g	acceleration of gravity
K	von Kármán's constant ($\cong 0.4$)
k	non-dimensional length of advective zone $= LRu^*/b^2U$
L	length of advective zone
q	mass inflow rate
R	hydraulic radius
R_x	flux rate
Ri	Richardson number
S	channel slope
t	time
t_0, t_1, t_2	times when certain events occur at sites x_0, x_1, x_2
u_x, u_y, u_z	velocity in x, y and z directions
U	cross section average velocity in x direction
u^*	shear velocity $=\sqrt{gRS}$
W	total mass input
x, y, z	distance in longitudinal, vertical and tranverse directions
x^*, y^*, z^*	non-dimensional distances
x_m	distance to attain complete vertical or transverse mixing
x_0, x_1, x_2, y_0, z_0	location of sites
$x_p{}^*$	location of peak concentration
x_s, y_s	maximum length and width in which concentration exceeds a specified level
ρ	density

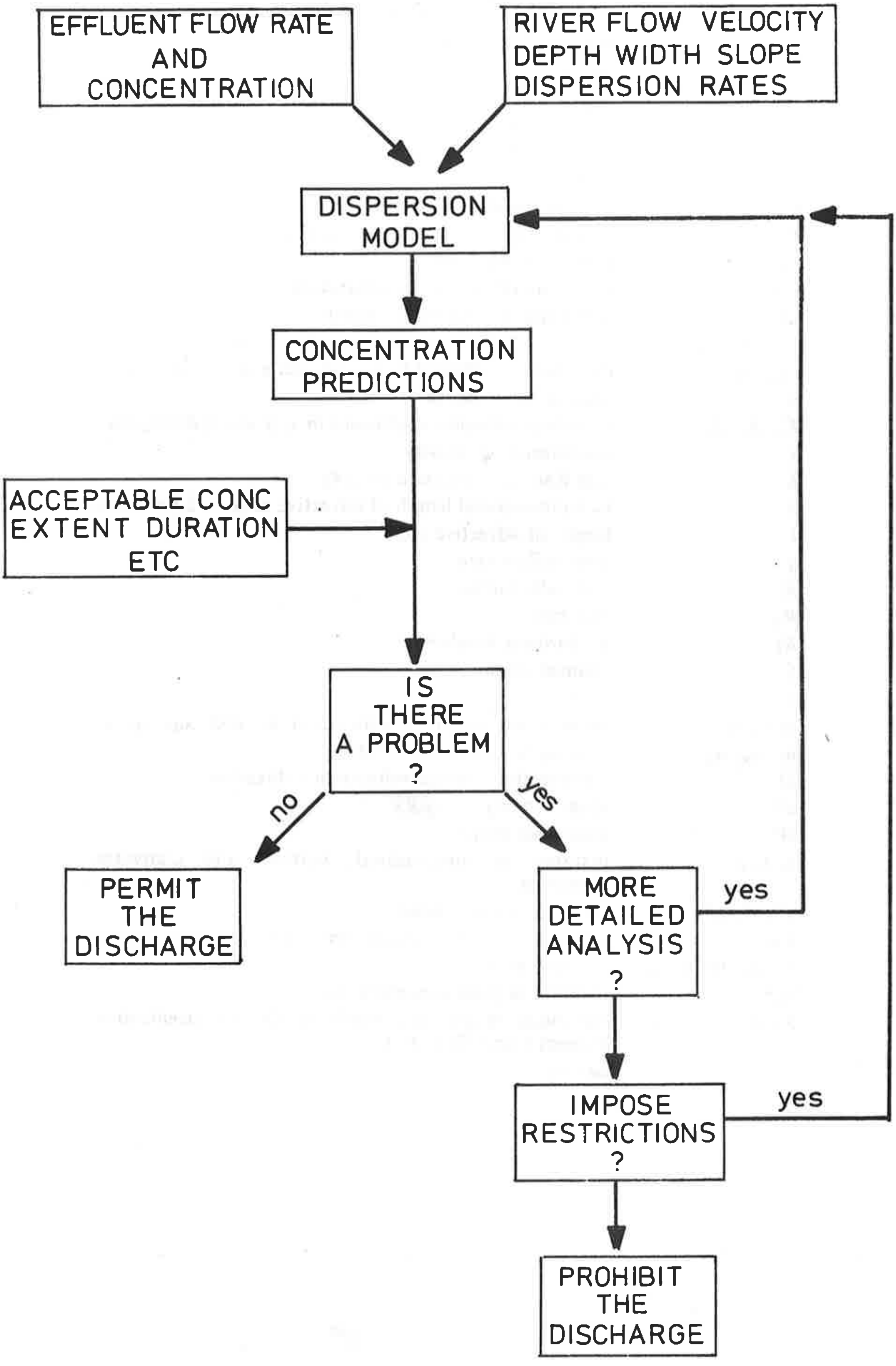

Figure 1.1 The steps and information required to assess the impact of effluents on water quality

1.0 INTRODUCTION

1.1 Scope of this handbook

One of the first steps when assessing the potential impact of an effluent on river water quality is to estimate resulting concentrations of potentially troublesome constituents. This requires knowledge of the velocities and rates of mixing in the receiving waterway. The mechanics of mixing in rivers are complex and have so far defied a complete mathematical description. There are, however, a number of semi-empirical techniques which can be used to analyse particular problems. It is the intention of this handbook to summarise simple techniques for predicting rates of mixing in rivers and to facilitate preliminary estimates of the impact of effluents on water quality.

Preliminary estimates may be sufficient to indicate whether or not an effluent will have an adverse effect on water quality, or they may indicate that further investigation, either experimental or theoretical, is justified. Figure 1.1 summarises the basic problem and the type of information required.

This handbook deals only with problems that can be solved conveniently using nomographs, programmable calculators or at most a small mini-computer. Large numerical models are not described because it is felt that these should not be employed in making a preliminary estimate of the impact of an effluent on water quality.

1.2 Mechanisms causing mixing in rivers

When material (hereinafter referred to as tracer for convenience) is discharged into a river two things happen to it. Firstly, it is carried away from the outfall by the current, a process which is termed advection; and secondly, it spreads out, a process which is termed dispersion.

In stagnant water and laminar flow, spreading is attributable to molecular motion and is called "molecular diffusion". The net transfer of tracer from a region of high concentration to a region of lower concentration proceeds at a rate proportional to the concentration gradient between the two regions. This is "Fick's Law" which in one dimension can be written mathematically

$$R_x = -D\frac{dC}{dx}$$

where R_x = transfer rate per unit area in the x direction, C = concentration, dC/dx = gradient in the x direction, and D = molecular diffusion coefficient, a constant.

In turbulent and non-uniform flow spreading proceeds at a much higher rate than in laminar flow. The reason for this is that velocity gradients act to increase concentration gradients and hence allow molecular diffusion to occur more rapidly. This is illustrated in Fig. 1.2. Such spreading is termed "dispersion" to distinguish it from "molecular diffusion". Strictly dispersion is still a molecular process but turbulence and velocity gradients greatly increase local concentration gradients and hence increase the rate at which tracer spreads.

In many situations the rate of dispersion can be approximated by Fick's Law. However, the value of D may be several orders of magnitude larger than for molecular diffusion and is highly variable. The variability arises partly because the size and intensity of turbulent eddies may vary considerably with position in the river channel, with changes in flow or location, and from one channel to another. For example the rate of dispersion can be expected to be smaller very close to the river bed (where velocity and intensity of turbulence may be small) than at mid depth. Also as the size of the tracer patch being investigated increases, the velocity gradients may change and larger eddies may become involved in mixing. Thus very close to an outfall the rate of dispersion can be expected to be smaller than it is further downstream.

1.3 Reducing the complexity of the problem

In the most general problem advection and dispersion will occur in each of the three co-ordinate directions, and the governing equations will be comparatively complex.

In many practical problems, however, the analysis can be simplified by neglecting terms which are small. This can be done: if any of the velocities is small, if any of the

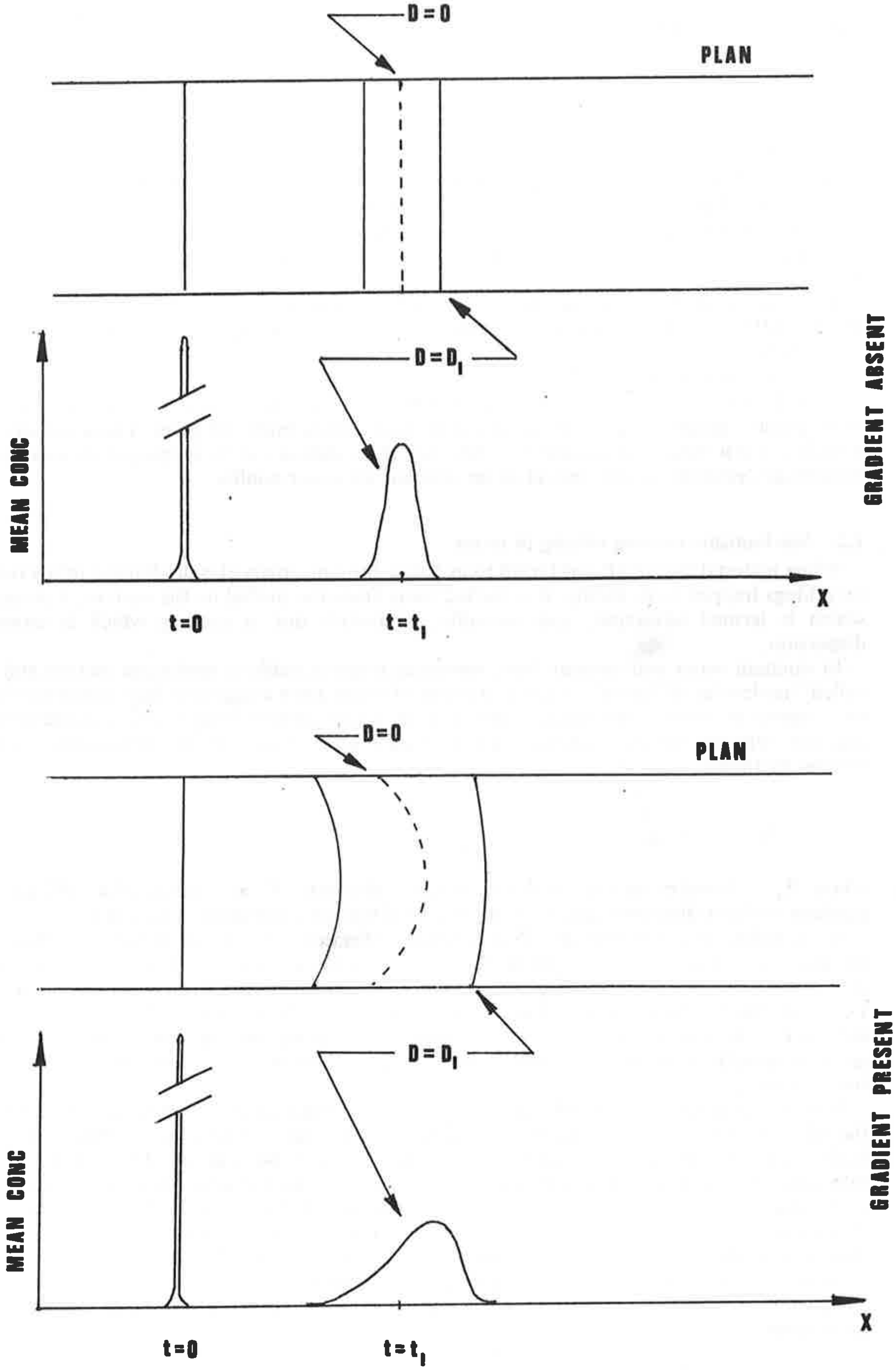

Figure 1.2 Sketch showing how a velocity gradient increases the dispersion rate.

concentration gradients is small because the tracer is far enough downstream from the outfall for the presence of channel boundaries to be felt, or if the nature of the discharge means that any concentration gradient is small.

In studying rivers we can make the following simplifications. Clearly vertical and lateral average velocities are small. Many rivers are wide but shallow, and tracer becomes well-mixed vertically before it becomes well-mixed transversely. Similarly tracer often becomes well-mixed transversely before it becomes well-mixed longitudinally. This means that vertical, transverse, and longitudinal mixing can sometimes be considered as separate one-dimensional problems. At other times longitudinal mixing can be neglected and the problem becomes two-dimensional in the vertical and transverse directions. Table 1.1 summarises the characteristics of various problems in river dispersion, and Fig. 1.3 illustrates three types of river dispersion problem.

1.4 Summary

(a) Tracer movement in a river comprises advection and dispersion.
(b) Advection is the net result of averaged velocities.
(c) Dispersion is the net result of molecular diffusion and non-uniformities in velocity.
(d) In many circumstances dispersion can be modelled approximately using Fick's Law.
(e) Although the general dispersion problem is three-dimensional, simplifications can sometimes be made to reduce the complexity of the problem.

Table 1.1 Summary of important dispersion problems in rivers and terms required to study them

Type of Source time	space (1)	(2)	Type of Solution (3)	Terms required (1) advection	dispersion	Number of dimensions
instantaneous	point source	near field	$0 < x < \frac{U}{2} d^2/D_y$	x (4)	x, y, z	3
		mid field	$\frac{U}{2} d^2/D_y < x < \frac{U}{2} b^2/D_z$	x	x, z	2
		far field	$\frac{U}{2} b^2/D_z < x < \infty$	x	x	1
steady	point source	near field	$0 < x < \frac{U}{2} d^2/D_y$	x (4)	y, z	3 (6)
		mid field	$\frac{U}{2} d^2/D_y < x < \frac{U}{2} b^2/D_z$	x	z	2 (6)
		far field	$\frac{U}{2} b^2/D_z < x < \infty$	– (5)	– (5)	0 (5)
instantaneous	z-line source	near field	$0 < x < \frac{U}{2} d^2/D_y$	x	x, y	2
		far field	$\frac{U}{2} d^2/D_y < x < \infty$	x	x	1
steady	z-line source	near field	$0 < x < \frac{U}{2} d^2/D_y$	x	y	2 (6)
		far field	$\frac{U}{2} d^2/D_y < x < \infty$	– (5)	– (5)	0 (5)
instantaneous	y-line source	near field	$0 < x < \frac{U}{2} b^2/D_z$	x	x, z	2
		far field	$\frac{U}{2} b^2/D_z < x < \infty$	x	x	1
steady	y-line source	near field	$0 < x < \frac{U}{2} b^2/D_z$	x	z	2 (6)
		far field	$\frac{U}{2} b^2/D_z < x < \infty$	– (5)	– (5)	0 (5)

NOTES:

(1) co-ordinate directions are shown in Fig. 1.3

(2) near field = very close to the outfall, mid field = moderately and far field = some considerable distance away

(3) D_x, D_y, D_z are dispersion coefficients and U = mean velocity (see equations 2.12 and 3.9 in text)

(4) on a very small scale, y, z advection may be present in the prototype

(5) concentration is constant (fully mixed)

(6) the dimensionality can be reduced by one if the co-ordinate system used travels downstream at mean velocity

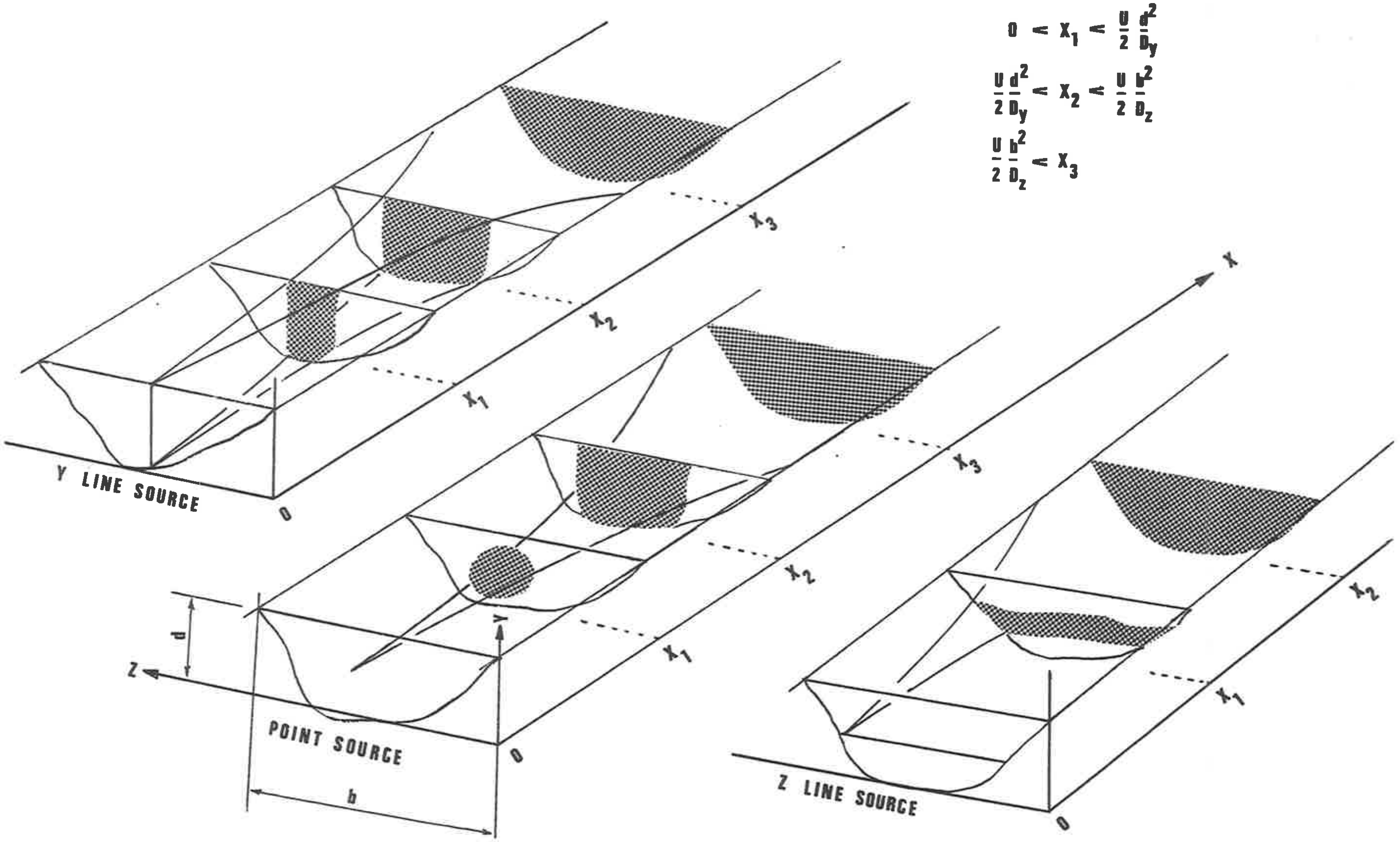

Figure 1.3 Sketch of three types of river dispersion problem.

2.0 VERTICAL MIXING

2.1 Mechanisms causing vertical mixing

In channels with no secondary circulations, the principal mechanism causing vertical mixing is turbulence generated by velocity shear. Theoretical work by Elder (1959) indicates that in such channels the dispersion coefficient varies parabolically with depth, and depends on both depth and shear velocity.

$$D_y(y) = \frac{y}{d}\left(1 - \frac{y}{d}\right) K\, d\, u^* \qquad (2.1)$$

where D_y = vertical dispersion coefficient, d = depth of flow, K = von Kármán's constant (= 0.4), and u^* = shear velocity = $\sqrt{gdS}$ where S = channel slope. This form has been confirmed in laboratory and field studies. For many practical problems the depth average is used (Fischer 1973).

$$D_y = 0.067\, d\, u^* \qquad (2.2)$$

Vertical secondary circulations can be expected to increase the rate of vertical mixing in natural channels. Few data are available to quantify their effect, but it appears that

$$0.067 < D_y/d\, u^* < 0.33 \qquad (2.3)$$

Table 2.1 summarises some reported values of D_y.

Table 2.1 Reported vertical dispersion coefficients

Reference	Channel	D_y $\text{cm}^2.\text{s}^{-1}$	D_y/du^* —	D_y/dU —
Elder (1959)	theoretical analysis	-	$\frac{y}{d}\left(1 - \frac{y}{d}\right)K$	-
Fischer (1973)	laboratory flume	-	0.067(1)	-
Fischer (1976)	James Estuary	-	-	2.9×10^{-4}(2)
Fischer (1976)	Kennebec Estuary	50–650	-	-
Fischer (1976)	Mersey River	5.71(3)	-	-
		500 (4)		

NOTES:
(1) depth mean value
(2) augmented by wind induced surface waves
(3) measured in stratified flow
(4) estimated for non-stratified flow

2.2 Effects of density stratification

In tidal channels density stratification often occurs with saline (more dense) water underlying fresh (less dense) water. In such stratification vertical water movement is suppressed by the density gradient and the coefficient of vertical dispersion is greatly reduced. Our understanding of the processes involved is poor and it is difficult to make accurate predictions of vertical mixing rates in stratified flow. The method outlined below must, therefore, be regarded as approximate.

The "strength" of the stratification is quantified by the non-dimensional gradient Richardson number, Ri, which is the ratio

$$\frac{\text{potential energy required for mixing}}{\text{kinetic energy available to cause mixing}}$$

$$Ri = g\,\frac{\partial\rho}{\partial y} / \rho\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2.4)$$

where g = acceleration of gravity, $\rho(y)$ = density, $u(y)$ = longitudinal velocity and y = depth.

The two gradients in equation 2.4 can be estimated satisfactorily from the slopes of straight lines fitted to density and velocity versus depth profiles measured in the field. In a tidal channel it is desirable to calculate average values of the gradients over the tidal period from, say, hourly measurements (see worked example 2.6.6).

An empirical relationship used to quantify the reduction in dispersion coefficient is

$$D_s = D_o\,(1 + a\,Ri)^b \tag{2.5}$$

where D_s and D_o = vertical dispersion coefficient in stratified and unstratified flow respectively and a and b = constants estimated variously as shown in Table 2.2.

Table 2.2 Coefficients describing the effects of stratification on the vertical dispersion coefficient (after Fischer 1976)

a	b
3.33	- 1.50
0.276	- 2.00
10	- 0.50

These models differ considerably at high Richardson numbers, (i.e. in highly stratified flow), but they indicate that vertical dispersion coefficients are reduced by a factor of 2–10 at $Ri = 1$ and 10–100 at $Ri = 10$. Clearly, therefore, the methods outlined here should only be used to make preliminary estimates of vertical mixing in stratified flow and field tests together with more detailed modelling should be undertaken to confirm findings.

2.3 Vertical mixing downstream from a steady uniform transverse line-source

The first problem considered is to predict tracer concentrations downstream from a steady uniform transverse line-source such as a perforated pipe which extends across the entire channel width. In this problem transverse concentration gradients are negligible because of the uniform line-source. Longitudinal gradients are also negligible because the source is steady. Thus the problem simplifies to become quasi one-dimensional (see Table 1.1). Since the analysis is only approximate, the velocity and dispersion coefficient are taken to be uniform over the depth as a rough approximation to turbulent flow. This simplification means that concentration estimates below outfalls on the river bed may be poorly estimated as explained below.

Figure 2.1 shows lines of equal concentration downstream from transverse line sources located at three different depths. Variables are expressed in non-dimensional form so that many combinations of parameters appear on the same graph.

$$C^* = C/\overline{C} = CUbd/q \tag{2.6}$$

$$y^* = y/d \tag{2.7}$$

$$x^* = x\,D_y/Ud^2 \tag{2.8}$$

where C^*, y^* and x^* = non-dimensional concentration, vertical displacement and downstream displacement respectively, C = concentration, $\overline{C}$ = fully mixed concentration, U = mean velocity, D_y = depth averaged vertical dispersion coefficient, d = river depth (the mean depth should be used here if the channel is irregular), b = river width and q = tracer mass inflow rate. The bed and water surface are located at $y^* = 0$ and $y^* = 1$ but the problem is symmetrical in y since the flow velocity is assumed uniform.

Clearly

$$0 < y^* < 1 \tag{2.9}$$

and

$$C^* = 1 \tag{2.10}$$

a long way below the outfall. The regions to the left of the $C^* = 0.001$ contour do not contain any tracer, while in the region to the right of the $C^* = 1.01$ and 0.99 contours the tracer is fully mixed.

Figure 2.1a may overestimate the rate of dispersion downstream from an outfall on the bed of a rough natural channel for the following reasons:

(i) the velocity very close to a boundary is small and hence concentrations will be higher locally than expected;

(ii) the value of the dispersion coefficient may be quite low very close to the boundary because the scale and intensity of turbulence are small (see equation 2.1);

(iii) irregularities in the bed, "dead zones", trap tracer and cause locally high concentrations.

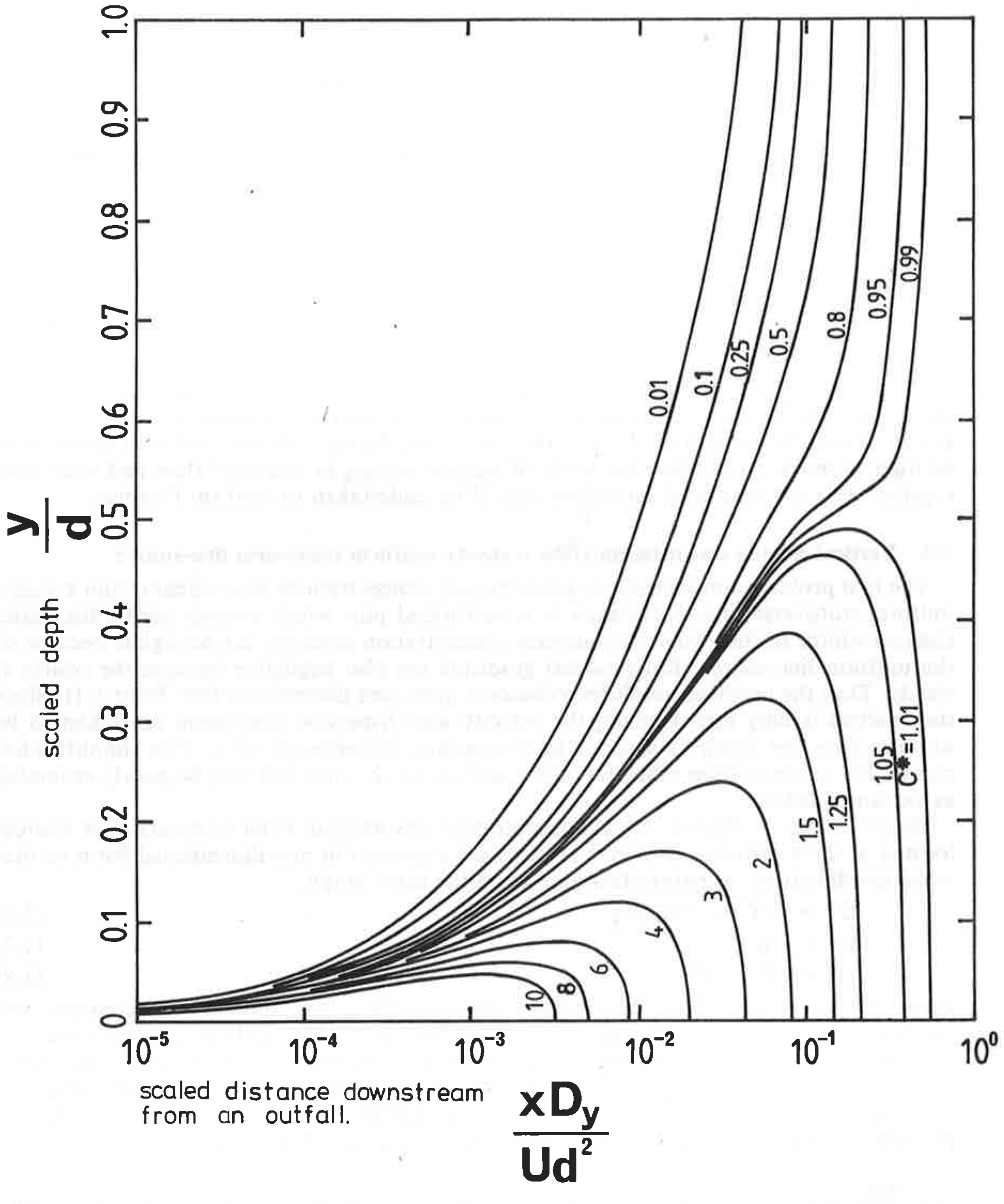

Figure 2.1a Concentration contours downstream from a steady transverse line source located on the channel bed. (See text for caveat on use of this figure; contours are of equal scaled concentration.)

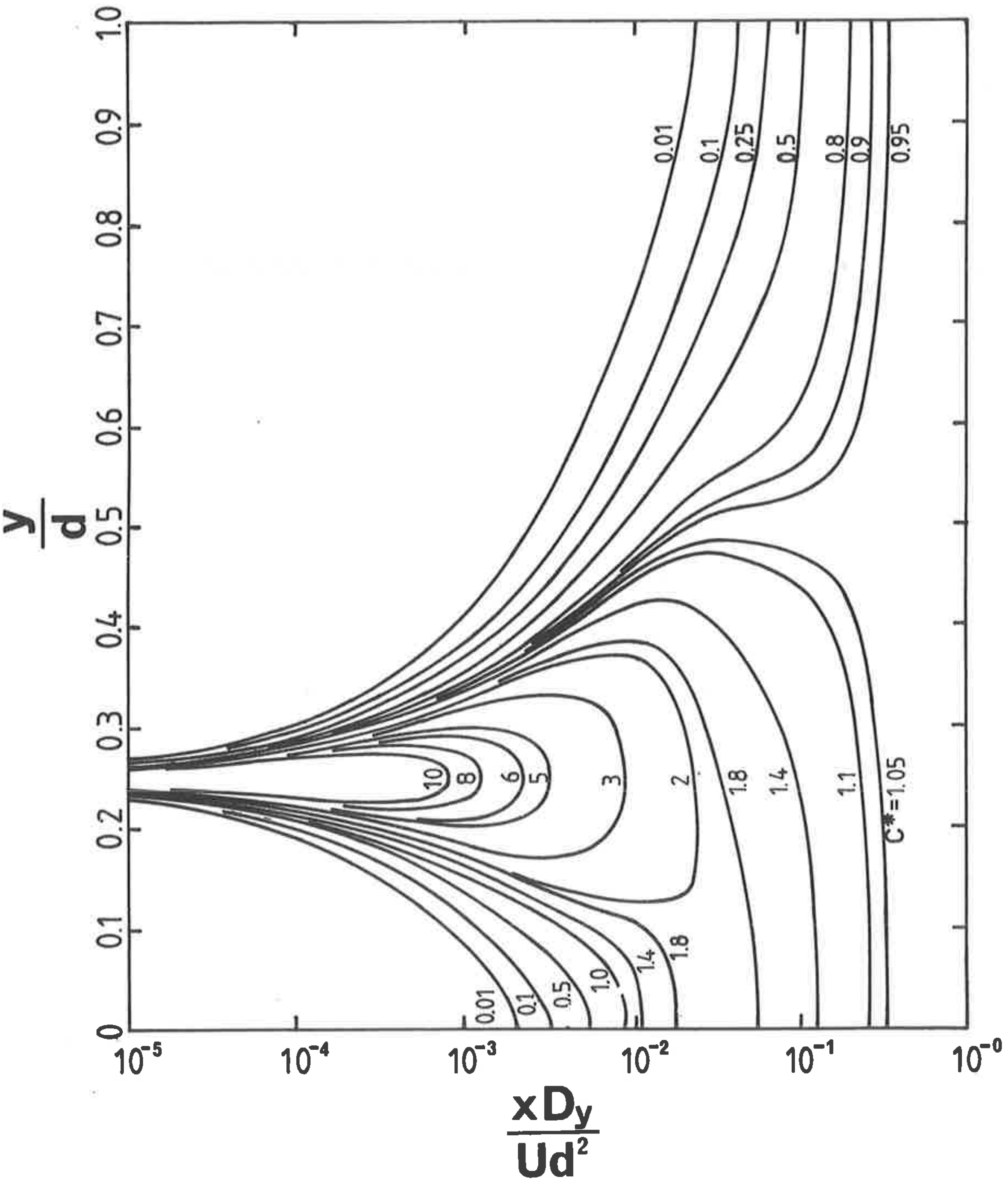

Figure 2.1b Concentration contours downstream from a steady transverse line source located at three-quarters depth.

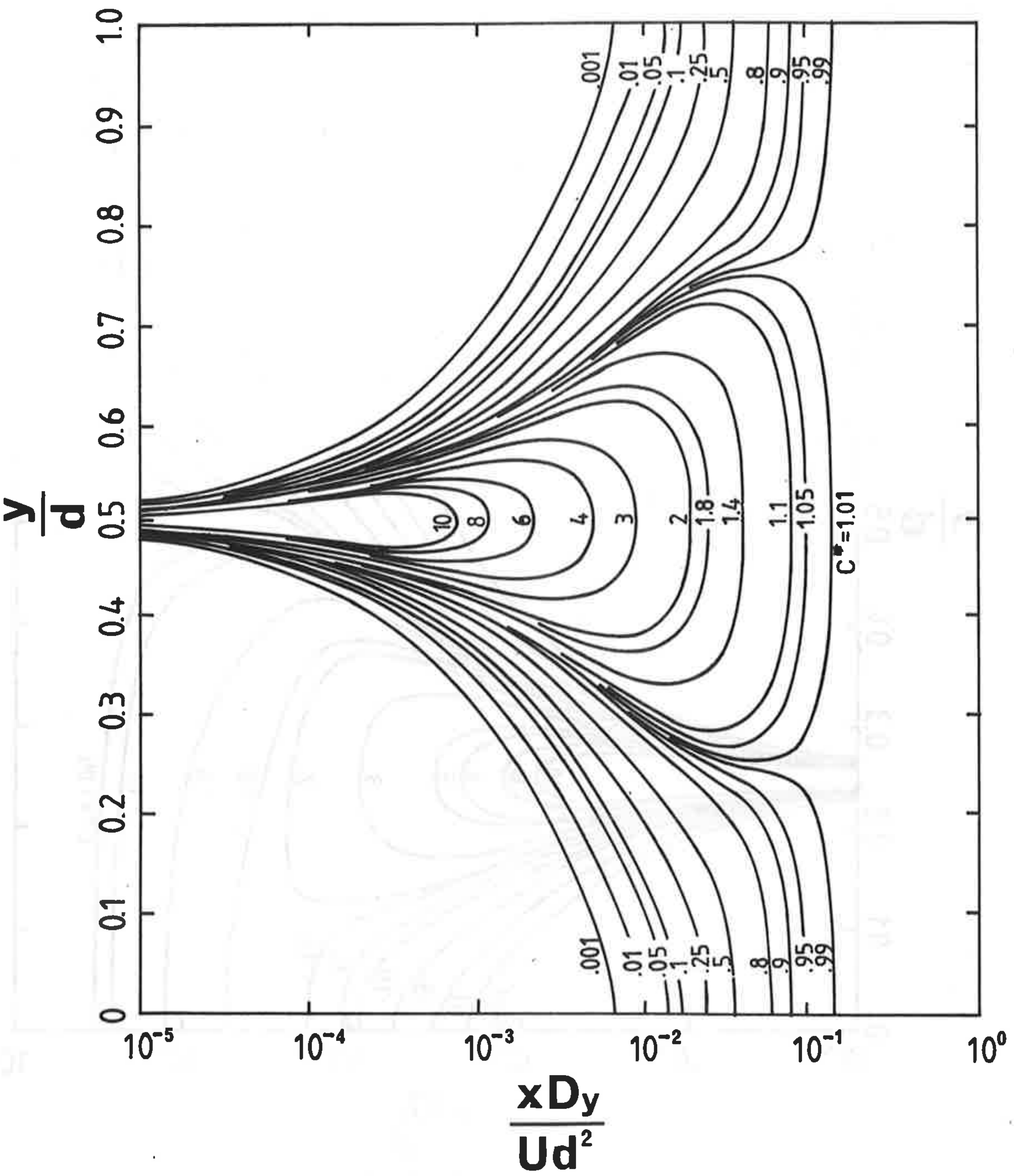

Figure 2.1c Concentration contours downstream from a steady transverse line source located at mid depth.